Representation of gray-scale windowed operators

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Introduction

- A fundamental problem in Mathematical Morphology is the design of function operators
- An approach for operators design is statistical optimization in a space of operators
- In the optimization, it is fixed a family of useful operators that have a standard representation
- The complexity of the optimization depends on the size of the family of operators considered

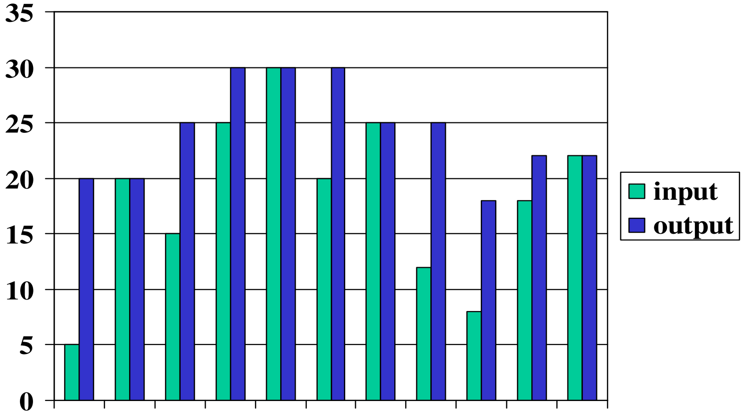
- In the binary case, the family of W-operators is usually considered
- The family of binary W-operators has $2^{2^{|W|}}$
- In the gray-scale case, the family of W-operators is also usually considered
- The family of gray-scale W-operators has $l^{m^{|W|}}$
- In ordinary applications $l=m=256$

- The family of WK-operators depends on a spatial window W and a gray-scale window K
- The family of WK-operators has $k^{k^{|W|}}$
- The complexity of the optimization problem is controlled by k and $|W|$
- The values of k and $|W|$ depends on the problem: $k=3$, 5, 7, ... and $|W| = 9, 25, 49, \ldots$

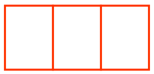
Characterization of W-operators

- Function translation: $f_x(z) = f(z - x)$
- Translation invariant operator : $\Psi(f_x) = \Psi(f)_x$
- Locally defined operator in *W*: $\Psi(f)(x) = \Psi(f / W_x)(x)$
- W-operator is t.i. and l.d.: $\Psi(f)(x) = \psi(f_{-x} / W)$

Dilation by W

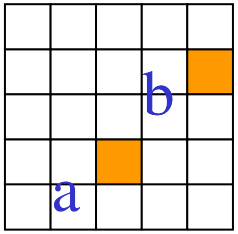


1	2	1	2
3	4	5	5
5	8	3	8
25	22	18	25

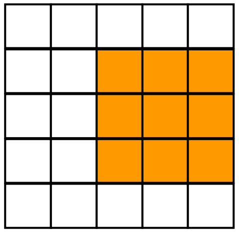
W 

Sup-representation of characteristic functions

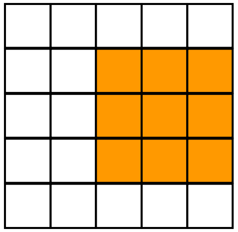
- Let a, $b \in$ Fun$[W,L]$, $a \leq b$ iff $a(x) \leq b(x)$, $x \in W$

|W| = 2 

- Interval $[a,b] = \{u \in \text{Fun}[W,L]: a \leq u \leq b\}$



- Sup-generating operator: $\lambda_{a,b}(u) = 1 \Leftrightarrow u \in [a,b]$



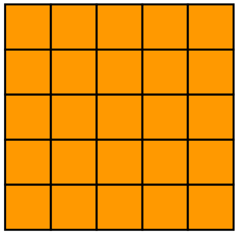
[a,b]

0	0	0	0	0
0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	0	0	0

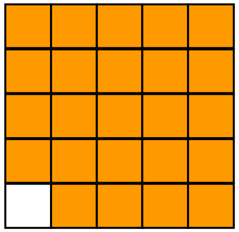
$\lambda_{a,b}$

Kernel of ψ at y: $K(\psi)(y) = \{u \in \mathrm{Fun}[W,L]: y \leq \psi(u)\}$

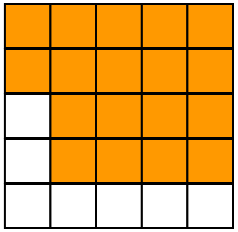
	-2	-1	0	1	2
2	0	1	2	2	2
1	0	1	2	2	2
0	-1	1	2	2	2
-1	-1	1	1	1	1
-2	-2	-1	-1	-1	-1



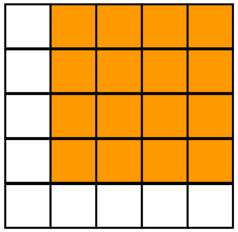
$K(\psi)(-2)$



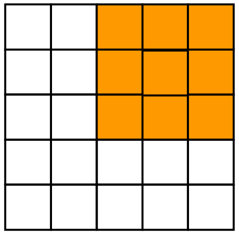
$K(\psi)(-1)$



$K(\psi)(0)$

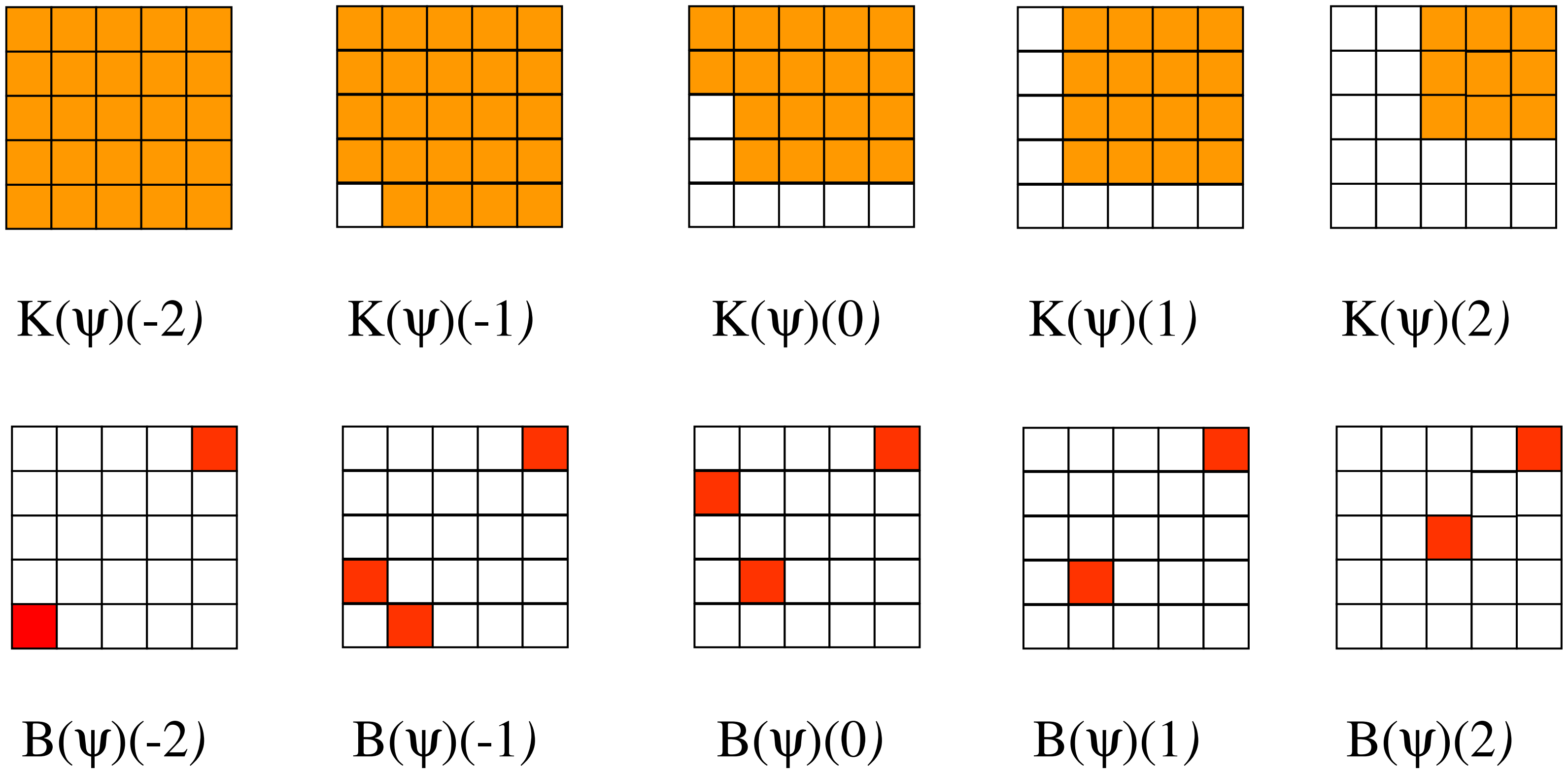


$K(\psi)(1)$



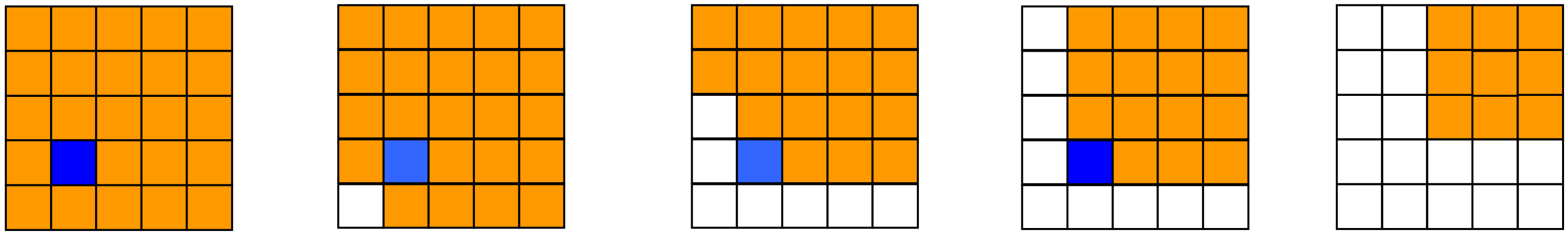
$K(\psi)(2)$

Basis of ψ at *y:* B(ψ) is the set of maximal intervals contained in K(ψ)



Sup-representation

$$\psi(u) = \bigcup\left\{y \in M : \bigcup\left\{\lambda_{a,b}(u) : [a,b] \in B(\psi)(y)\right\} = 1\right\}$$



$K(\psi)(-2)$ $K(\psi)(-1)$ $K(\psi)(0)$ $K(\psi)(1)$ $K(\psi)(2)$

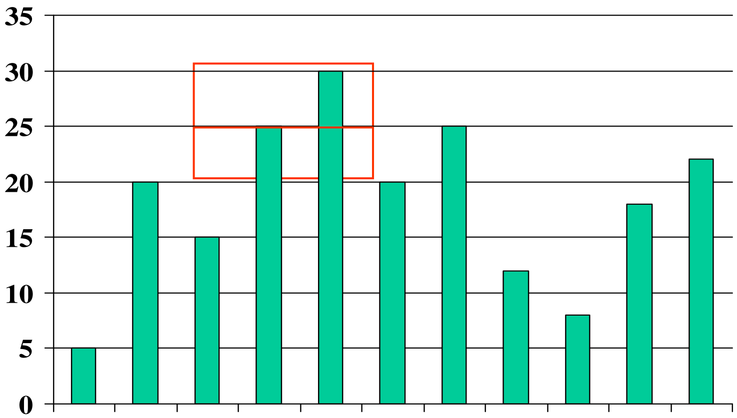
$\psi(-1,-1) = 1$

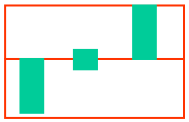
K-characteristic functions

- Gray-scale translation: $(u + y)(z) = u(z) + y$
- Gray-scale window: $\left\{-\frac{k-1}{2},\ldots,-1,0,1,\ldots,\frac{k-1}{2}\right\}$

Windowing of u at y

$$(u / K_y)(z) = \bigcap \left\{ \bigcup \left\{ -\frac{k-1}{2}, u(z) - y \right\}, \frac{k-1}{2} \right\}$$





- gray-scale t. i.: $\psi(u+y)=\psi(u)+y$
- locally defined in *K*: $\psi(u)=u(o)+\beta_{u(o)}\left(u/K_{u(o)}\right)$
- Representation: $\psi(u)=u(o)+\beta_{\psi}\left(u/K_{u(o)}\right)$

β_ψ

2	-2	1	2	2	2
1	-2	1	2	2	2
0	-2	1	2	2	2
-1	-2	1	1	1	1
-2	-2	-2	-2	-2	-2
	-2	-1	0	1	2

ψ

14	12	13	14	15	16
13	12	13	14	15	15
12	12	13	14	14	12
11	12	13	13	11	12
10	12	12	10	11	12
	10	11	12	13	14

=

$u(o)$

14	10	11	12	13	14
13	10	11	12	13	14
12	10	11	12	13	14
11	10	11	12	13	14
10	10	11	12	13	14
	10	11	12	13	14

+

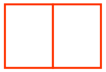
β_ψ

14	2	2	2	2	2
13	2	2	2	2	1
12	2	2	2	1	-2
11	2	2	1	-2	-2
10	2	1	-2	-2	-2
	10	11	12	13	14

WK-operators

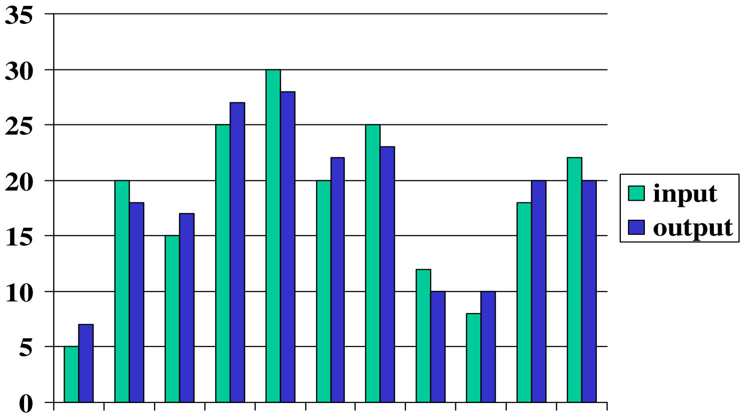
β_ψ

	-2	-1	0	1	2
2	-2	1	2	2	2
1	-2	1	2	2	2
0	-2	1	2	2	2
-1	-2	1	1	1	1
-2	-2	-2	-2	-2	-2

W 

$$K = \{-2,-1,0,1,2\}$$

Ψ



Conclusion

- The family of WK-operators was introduced
- The size of the family of WK-operators depends on K and $|W|$
- A WK-operator is characterized by a computational function that has a sup-representation
- Preliminary results with design of WK-operators are encouraging
- This is a general approach for designing Image or Signal Processing operators