MAC0329 - Álgebra Booleana e Aplicações

Lista de Exercícios 1

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1. Give the graphical representation of the relation α from $A \equiv \{1, 2, 3\}$ to $B \equiv \{a, b, c\}$.

$$R_\alpha \equiv \{(1, a), (1, b), (2, a), (2, c), (3, a), (3, b), (3, c)\}$$

2. Show that for any binary relations α and β from A to B, we have

   a) $(\alpha^{-1})^{-1} = \alpha$

   b) $\alpha^{-1} = \beta^{-1}$ if and only if $\alpha = \beta$

   c) $\alpha^{-1} \subseteq \beta^{-1}$ if and only if $\alpha \subseteq \beta$

3. Define the complement of a relation α, denoted $\bar{\alpha}$, as the relation such that $a\,\bar{\alpha}\,b$ if and only if $(a, b) \notin R_\alpha$. Show that $(\bar{\alpha})^{-1} = \overline{(\alpha)^{-1}}$.

4. Determine which one of the following sets defines a function.

   a) $\{(i, i^2) : i$ is integer $\geq 1\}$.

   b) $\{(a, (b, c)), (a, (c, b)), (b, (c, d))\}$ for $A \equiv \{a, b, c, d\}$.

5. Determine which one of the following functions satisfies the properties of being one-to-one (*injection*) or onto (*surjection*).

   a) Let f be a function from P, the set of positive integers to itself such that, for each $n \in P$, $f(n) = n + 1$. (f is called *successor function*)

   b) Let f be a function from the set $K \equiv \{1, 2, \ldots, k\}$ of positive integers less than or equal to k to itself such that

$$f(i) = \begin{cases} i + 1, & \text{for } 1 \leq i < k \\ 1, & i = k. \end{cases}$$

   c) Let $f : P \to \{0, 1, 2, 3\}$ such that for each $i \in P$,

$$f(i) = \begin{cases} 0, & \text{if } i \text{ is divisible by } 3 \\ 1, & \text{if } i \text{ is divisible by } 7 \\ 2, & \text{if } i \text{ is divisible by } 21 \\ 3, & \text{otherwise.} \end{cases}$$

6. Let $f : A \to B$ be a given function and $A' \subseteq A$, $B' \subseteq B$. Show that

a) $A' \subseteq f^{-1}(f(A'))$

b) $f(f^{-1}(B')) = B'$

where $f^{-1}(B') = \{a \,:\, a \in A \text{ and } f(a) \in B'\}$. (Recall that f^{-1} is the inverse binary relation of f, considered as a binary relation.)

7. Determine whether each of the following relations, defined by ordered pairs (m, n) of positive integers, has each of the properties: reflexivity, symmetry, antisymmetry and transitivity.

a) m is divisible by n	b) $m + n \geq 50$
c) $m + n$ is even	d) $m + n$ is odd
e) mn is even	f) m is a power of n
g) $m + n$ is a multiple of 3	h) m is greater than n

8. Let α be a relation on a set A. Is the relation defined by $R_\alpha \cap R_{\alpha^{-1}}$ symmetric?

9. A relation α on A is said to be *irreflexive* if for eah $a \in A$, $(a, a) \notin R_\alpha$. Show that a nonempty symmetric and transitive relation cannot be irreflexive.

10. Determine which of the following sets define an equivalence relation and which define a compatibility relation.

    a) $A \equiv \{(x, y) : x \text{ and } y \text{ are people of the same age}\}$

    b) $B \equiv \{(x, y) : x \text{ and } y \text{ are cousins}\}$

11. a) Which of the following tables of binary operations on the set $\{1, 2\}$ is associative?

	1	2
1	1	1
2	1	2

	1	2
1	1	1
2	2	2

	1	2
1	1	2
2	1	1

	1	2
1	1	2
2	2	2

	1	2
1	2	1
2	1	2

    b) Which of these operations is commutative?

12. For each of the following binary operations $\star$, determine whether it is associative or commutative.

    a) On the integers, $a \star b = a - b$.

    b) On the positive integers, $a \star b = a^b$.

    c) On the integers, $a \star b = ab + 5$.