MAC0329 - Álgebra Booleana e Aplicações

Lista de Exercícios 2

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1. Determine which of the following relations is a partial ordering.

   a) The relation "less than or equal to" on the set of integers.

   b) The relation "is a subset of" in a collection of sets.

   c) The relation "is a proper subset of" in a collection of sets.

   d) The lexicographic ordering of the English words.

2. Draw the Hasse diagram, and list all minimal and maximal elements and the least and greatest elements (if they exist) of each of the following posets.

   a) $[\mathcal{P}(S), \subseteq]$, where $S \equiv \{1, 2\}$ and $\mathcal{P}(S)$ denote the set of all subsets of S

   b) $[S, |]$, where $S \equiv \{1, 2, 3, 5, 10, 15, 30\}$ and $|$ is the relation "to be divisible by"

3. Let $[S, \leq]$ be a poset, and let a, b be two elements of S such that $a \leq b$. The set $\{x : a \leq x \leq b\}$ is called *interval* of S, denoted by $[a, b]$. Show that $[a, b]$ is a poset containing a least element and a greates element. Furthermore, show that $[a, b] = [c, d]$ if and only if $a = c$ and $b = d$.

4. A subset I of a lattice L is called an *ideal* of L if I satisfies the two following conditions:

   a) $a, b \in I$ implies that $a \vee b \in I$;

   b) for any element $x \in L$ and $a \in I$, $a \wedge x \in I$.

   Show that every ideal is a sublattice of L.

5. A poset is a $[S, \leq]$ is a *join-semilattice* (meet-semilattice) if for arbitrary elements a, b in S, $a \vee b$ ($a \wedge b$) exists and it is unique.
   Show that a poset is a lattice if and only if it is both a join- and a meet-semilattice.

6. Determine which of the following lattices are distributive.

   a) A chain (*simply* –or *linearly*– *ordered set*, that is, those posets for which any pair of elements is comparable).

   b) All subsets of a set under union and intersection.

   d) The dual of a distributive lattice.

   e) An arbitrary sublattice of a distributive lattice.

7. Add two new elements O' and I' to a distributive lattice L such that $O' \leq x \leq I'$ for every $x \in L$. Show that the resulting lattice is still distributive.

8. Consider the distributive lattice consisting of the set $\{1, 2, 3, 5, 6, 10, 15, 30\}$ under the partial ordering of divisibility.

   a) Is this lattice distributive?

   b) Find all of its join-irreducible elements.

   c) Express 15 as a join of join-irreducible elements.

9. a) Define the concept of a meet-irreducible element of a lattice and meet representation of elements of a lattice.

   b) Find all meet irreducible elements of the lattice given in exercise 9.

   c) Express the element 3 as an irreduntant meet of meet-irreducible elements in the lattice given in exercise 9.