Álgebra Booleana

1. Symbolize the following statement and determine its truth values.
   If it is hot in Arizona and it is raining outside or demonstrators are in the streets, then it is hot in Arizona, demonstrators are in the streets, and it is snowing in Argentina.

2. Suppose three one-digit binary number a, b and c are to be added together to form a two-digit number whose digits are denoted by s1s2. For each of the above five binary digits, define corresponding capital letter to be a statement variable which is True whenever the small letter digit is 1 and False otherwise. Determine a truth functional expression for s2 such that s2 is true whenever s2=1. Determine a similar expression for s1. Construct the diagram of logic circuits realizing s2 and s1.

3. A logic circuit is to be designed having 4 inputs y1, y0, x1 and x0. The pair of bits y1y0 and x1x0 represent 2-bit binary numbers with y1 and x1 as the most significant bits. The only circuit output, z, is to be 1 if and only if the binary number x1x0 is greater than or equal to the binary number y1y0. Determine a minimal sum of products expression for z.

4. A prime number is a number which is only divisible by itself and 1. Suppose the numbers between 0 and 31 are represented in binary in the form of the five bits

   x4x3x2x1x0

   where x4 is the most significant bit. Design a primer detector. That is, design a combinatorial logic circuit whose output, z, will be 1 if and only if the 5 input bits represent a prime number. Do not count 0 as a prime. Base your design on obtaining a minimal two-level representation.

5. Let the dual of a Boolean function be denoted f^*. Prove the theorem:

$$f^*(x_1, x_2) = \bar{f}(\bar{x}_1, \bar{x}_2).$$

6. A Boolean functions is self-dual if the dual of this function is equal to the function itself ($f = f^*$). Show that there are 2^{2n-1} self-dual functions of n variables.

7. Prove that

$$f(x_1, x_2, ..., x_n) = x_1 f(1, x_2, ..., x_n) + \bar{x}_1\, f(0, x_2, ..., x_n)$$

8. Determine a minimal sum of products (product of sums) realization for the following incompletely specified functions:

(a) f(A,B,C,D) = ∑ m(1,3,5,8,9,11,15) + d(2,13)
(b) f(W,X,Y,Z) = ∑ m(4,5,7,12,14,15) + d(3,8,10)
(c) f(A,B,C,D,E) = ∑ m(1,2,3,4,5,11,18,19,20,21,23,28,31) + d(0,12,15,27,30)
(d) A,B,C,D,E) = ∑ m(7,8,9,12,13,14,19,23,24,27,,29,30) + d(1,10,17,26,28,31)
(e) F(U,V,W,X,Y,Z) = ∑ m(0,2,14,18,21,27,32,41,49,53,62) + d(6,9,25,34,55,57,61)