Mudança de Estrutura de Representação de Operadores em Morfologia Matemática

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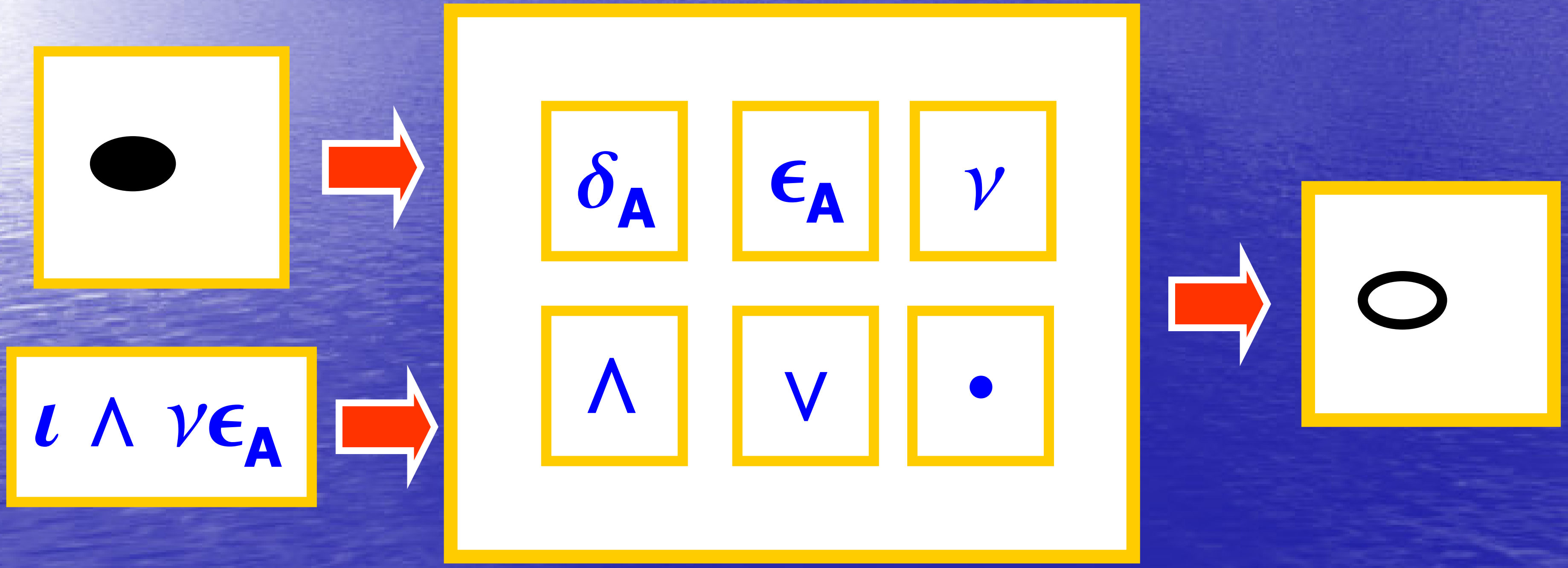
Agosto de 2000

Índice

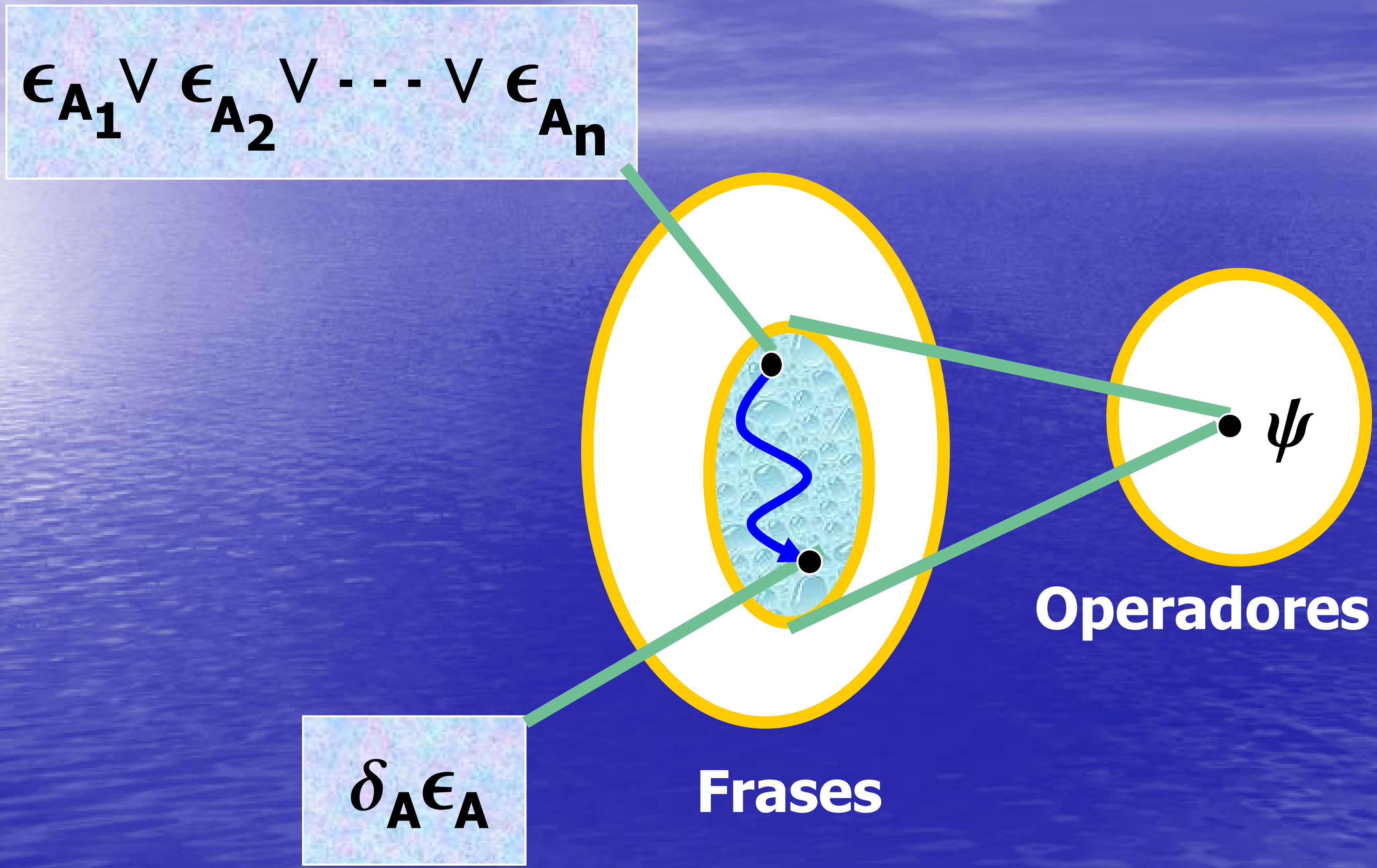
- Introdução
- Decomposição de EE's
  - EE's Convexos
  - EE's Simplesmente Conexos
  - EE's Arbitrários
- Representação Compacta
- Representação Seqüencial

Introdução

Máquina Morfológica



Introdução



$\epsilon_{A_1} \vee \epsilon_{A_2} \vee \cdots \vee \epsilon_{A_n}$
ψ
Operadores
$\delta_A \epsilon_A$
Frases

Introdução

Problema Geral :

Base de ψ

Uma representação equivalente de ψ

Implementação eficiente de ψ

Introdução

- Decomposição de EE´s
- Representação Compacta
- Representação Seqüencial

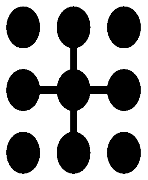
Mudança de Estrutura

Decomposição de EE´s

Decomposição de EE´s

Dado um EE A

Encontrar uma seqüência $[B_1, B_2, \cdots, B_n]$

$B_i \subseteq$ 

$A = B_1 \oplus B_2 \oplus \cdots \oplus B_n$

n mínimo

Decomposição de EE´s

- EE´s Convexos
- EE´s Simplesmente Conexos
- EE´s Arbitrários

Decomposição de EE´s

EE´s Convexos

EE´s Convexos

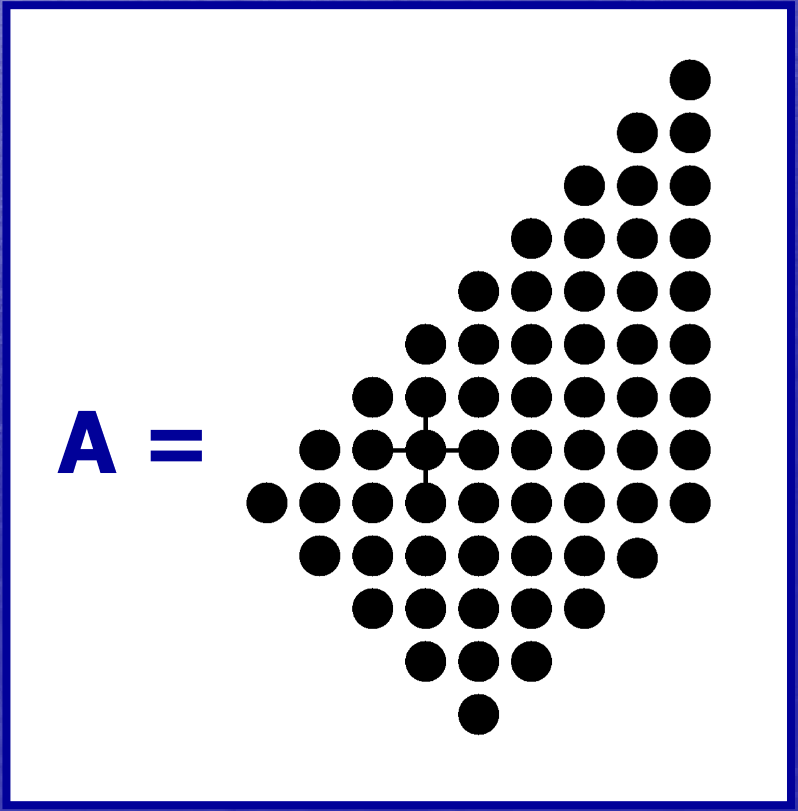
J. Xu

Decomposition of Convex Polygonal Morphological Structuring Elements into Neighborhood Subsets

1991

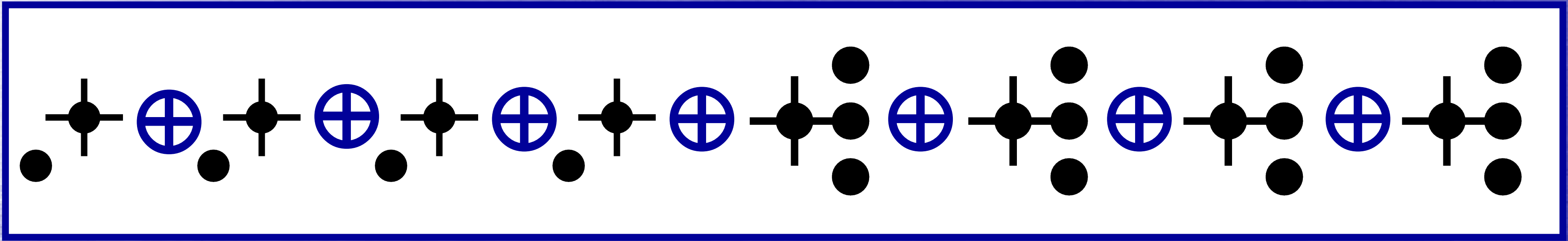
EE´s Convexos

Algoritmo do Xu

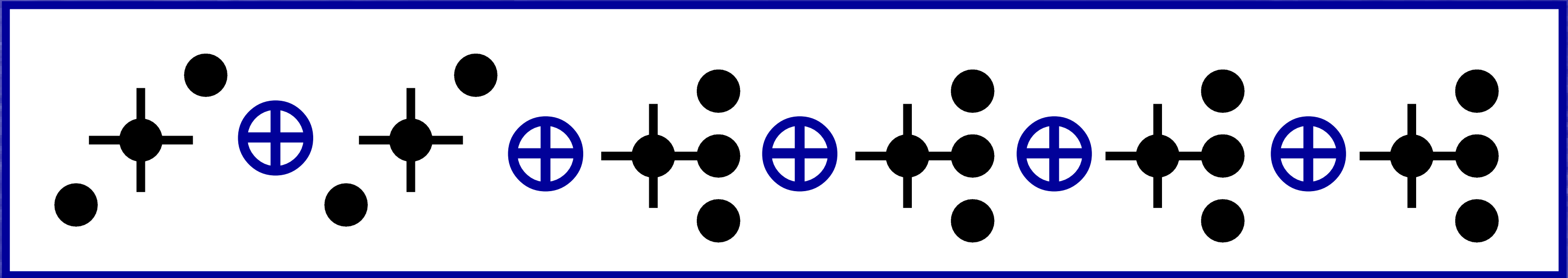


EE´s Convexos

Encontra uma decomposição

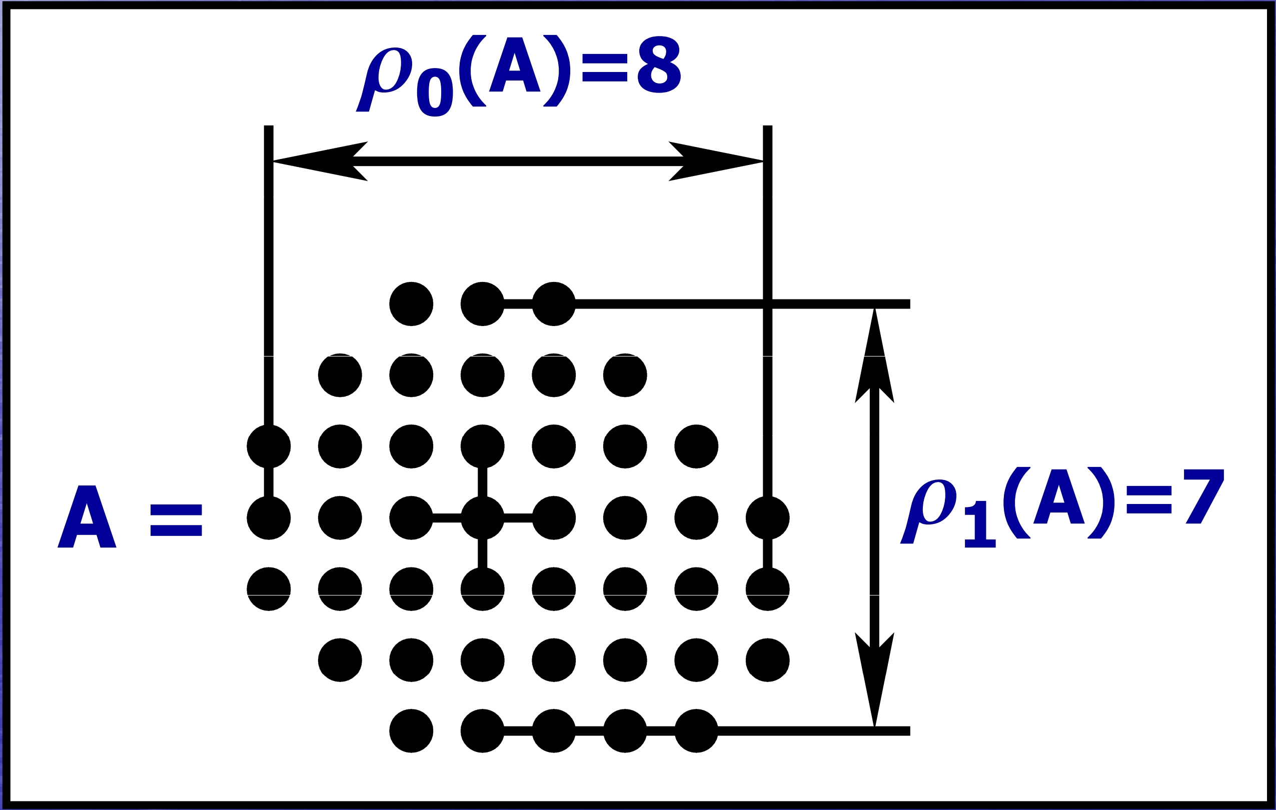


Aplica um processo de otimização



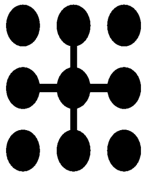
EE´s Convexos

Vetor Retangular



EE´s Convexos

$$SeqQ=[B_1, B_2, \cdots, B_n]$$

$B_i \subseteq$ 

$$\rho_0(B_i)+\rho_1(B_i) \geq \rho_0(B_k)+\rho_1(B_k) \quad i < k$$

$$\rho_0(B_i)+\rho_1(B_i) = \rho_0(B_k)+\rho_1(B_k) \Rightarrow |B_i| \leq |B_k|$$

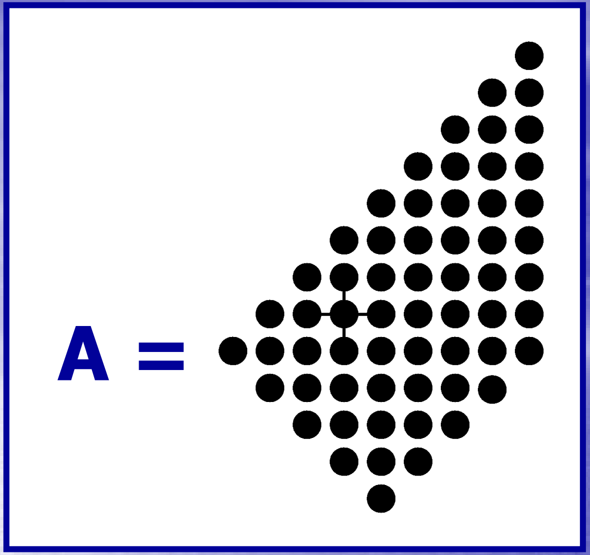
Decomposição de EE´s

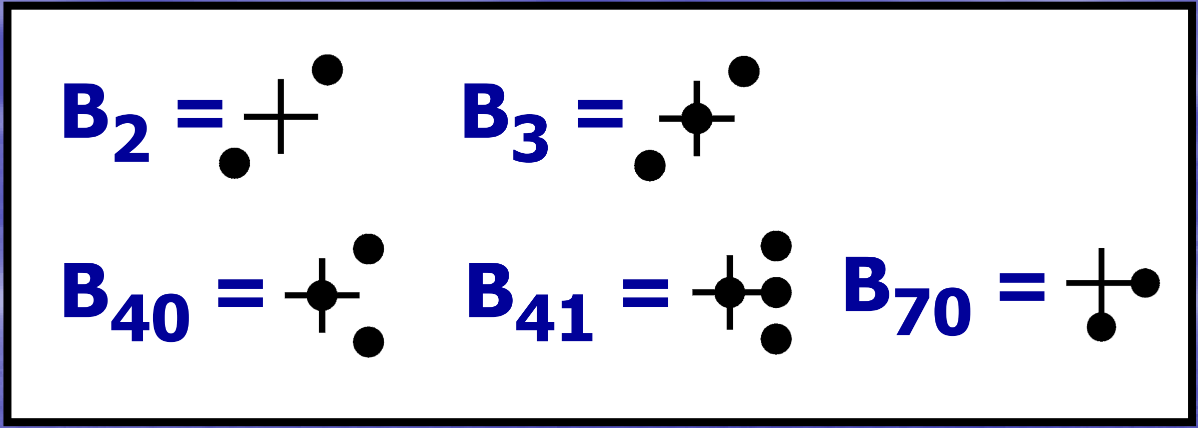
Invariante de um EE

B é invariante de A

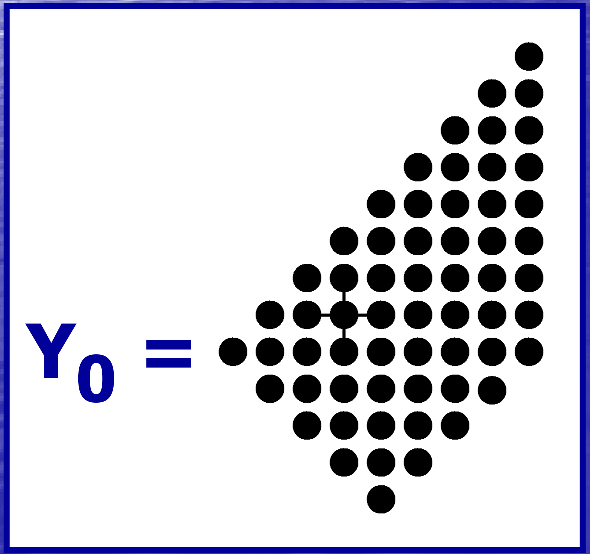
$\Updownarrow$

$$B = (A \ominus B) \oplus B$$





$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

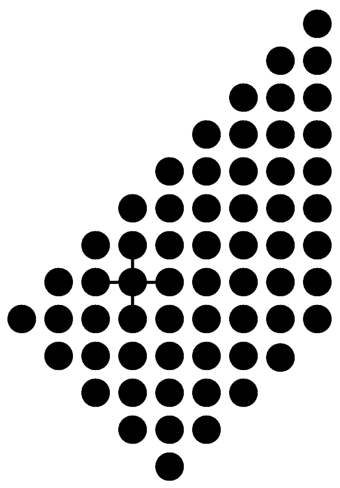


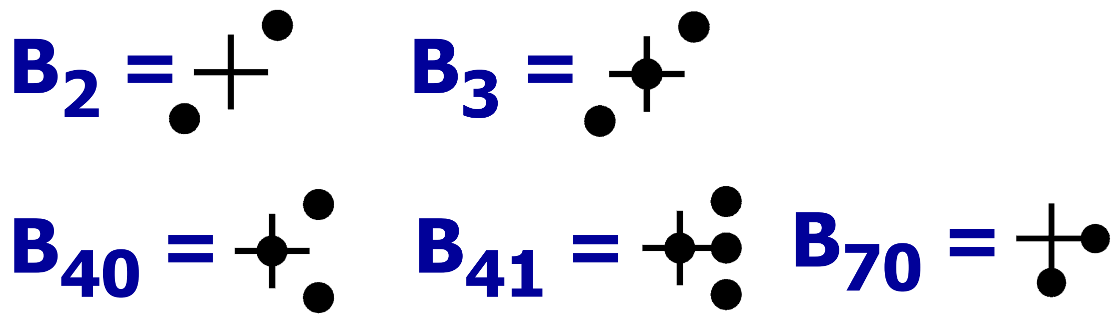
$S_0[A]=[\ ]$

$\rho(Y_0) = (8,12)$

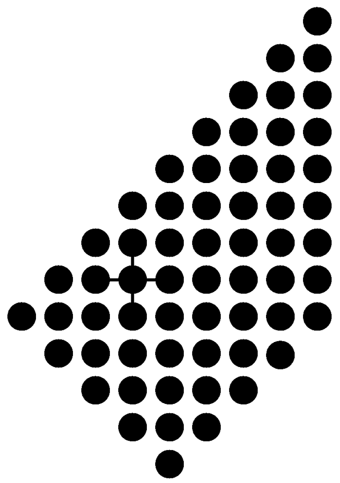
$\rho(Y_0) \neq (0,0)$? **(Sim)**

B_1 é invariante de Y_0? **(não)**

$A =$ 



$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

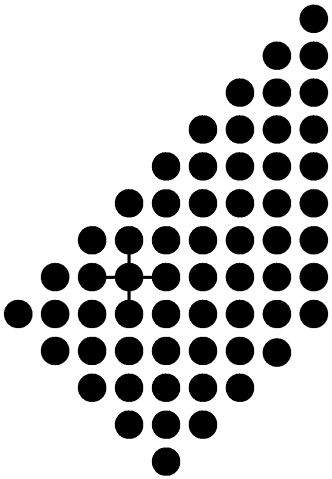
$Y_0 =$ 

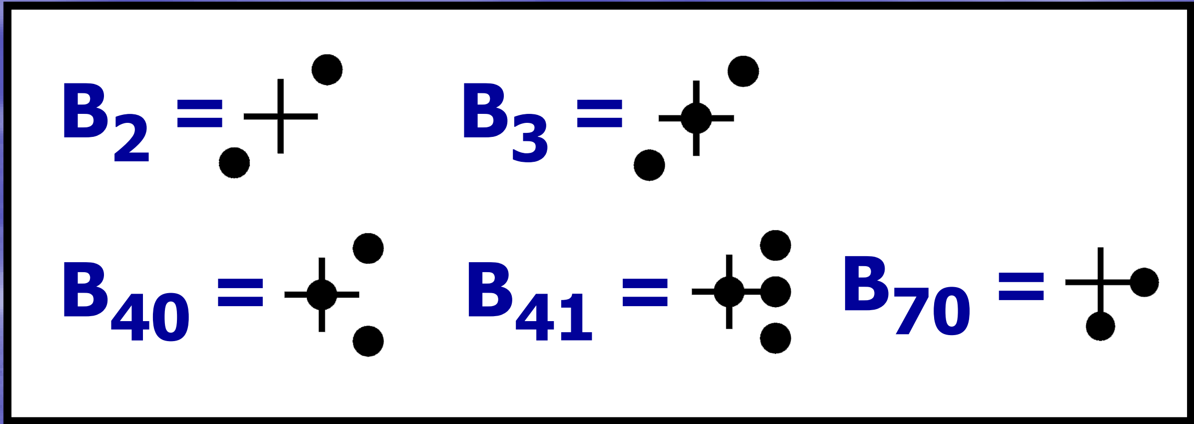
$S_0[A]=[\ ]$

$\rho(Y_0) = (8,12)$

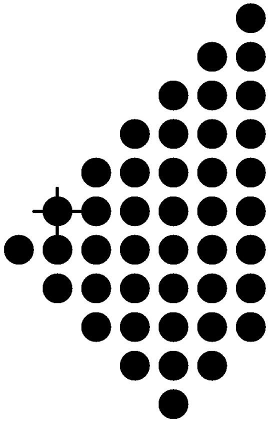
$\rho(Y_0) \neq (0,0)$? **(Sim)**

B_2 é invariante de Y_0? **(Sim)**

$A =$ 



$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

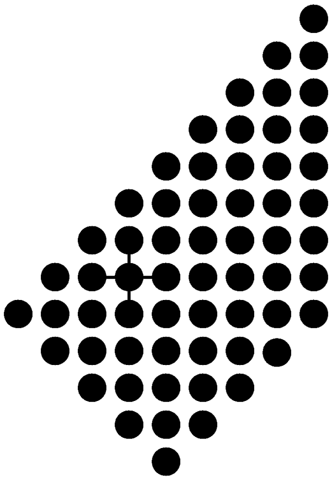
$Y_1 =$ 

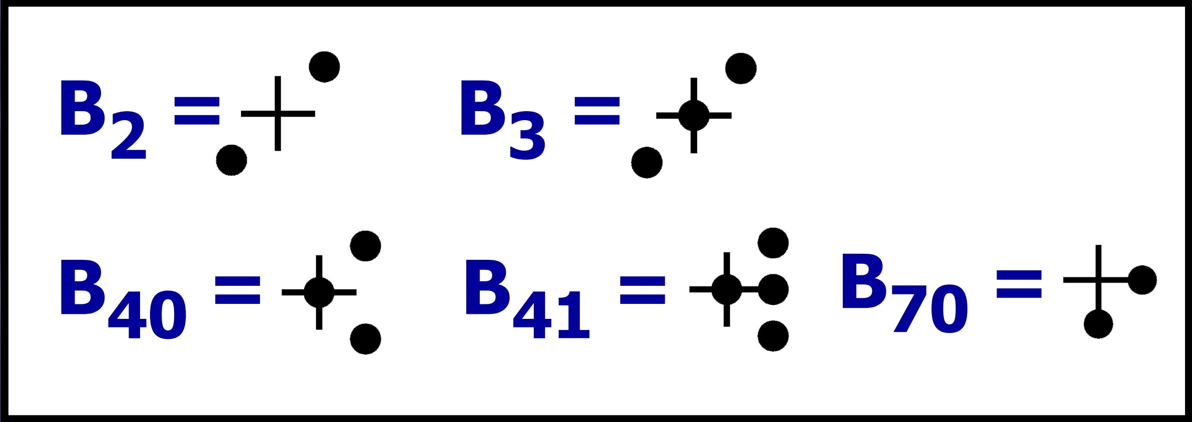
$S_1[A]=[B_2]$

$\rho(Y_1) = (6,10)$

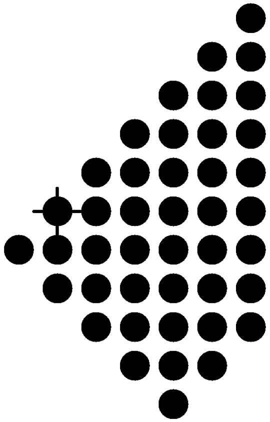
$\rho(Y_1) \neq (0,0)$? **(Sim)**

B_2 é invariante de Y_1? **(Não)**

$A =$ 



$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

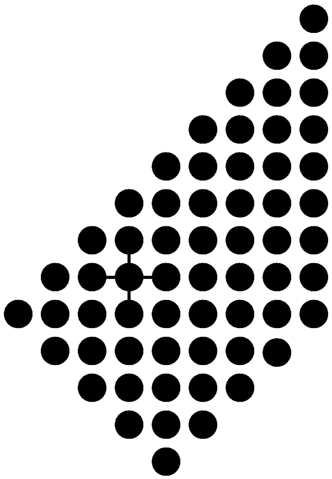
$Y_1 =$ 

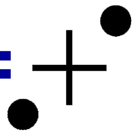
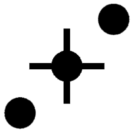
$S_1[A]=[B_2]$

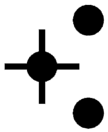
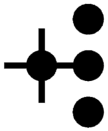
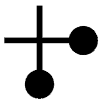
$\rho(Y_1) = (6,10)$

$\rho(Y_1) \neq (0,0)$? **(Sim)**

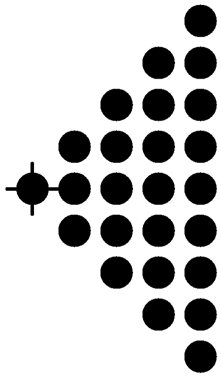
B_3 é invariante de Y_1? **(Sim)**

A = 

B_2 =  B_3 = 

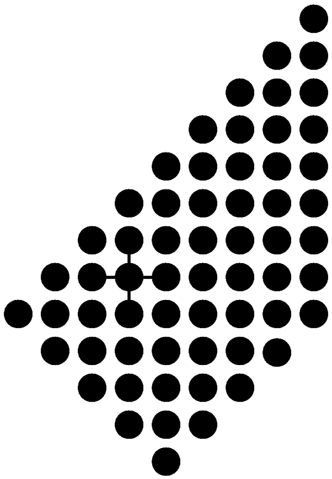
B_{40} =  B_{41} =  B_{70} = 

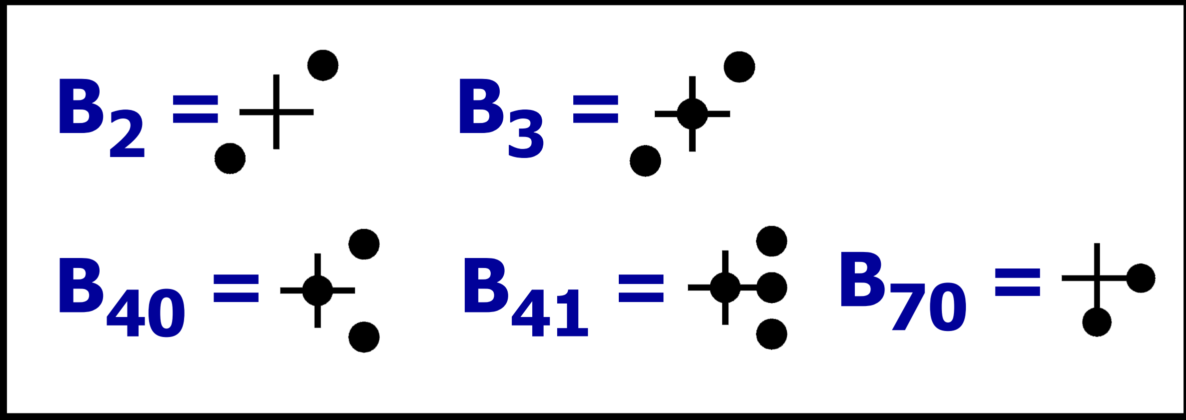
SeqQ=[B_1, B_2, B_3, - - - , B_{40}, B_{41}, - - - ,B_{70} , - - -]

Y_2 = 

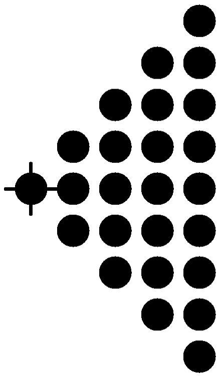
$S_2[A]=[B_2, B_3]$
$\rho(Y_2) = (4,8)$

$\rho(Y_2) \neq (0,0)$? **(Sim)**
B_3 é invariante de Y_2? **(Não)**

A =




SeqQ=[B_1, B_2, B_3, - - - , B_{40}, B_{41}, - - - ,B_{70} , - - -]

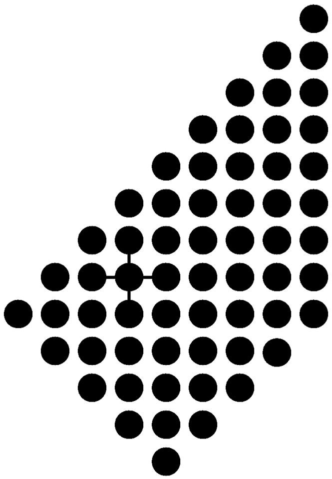
Y_2 =


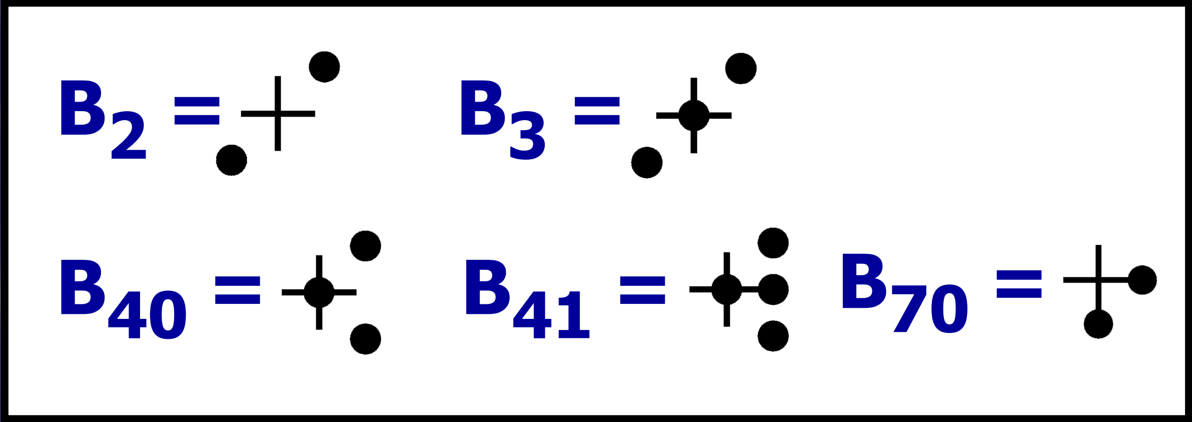
$S_2[A]=[B_2, B_3]$

$\rho(Y_2) = (4,8)$

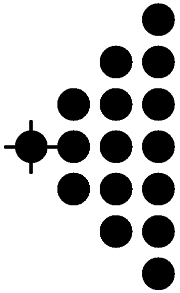
$\rho(Y_2) \neq (0,0)$? (Sim)

B_{40} é invariante de Y_2? (Sim)

A = 



$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

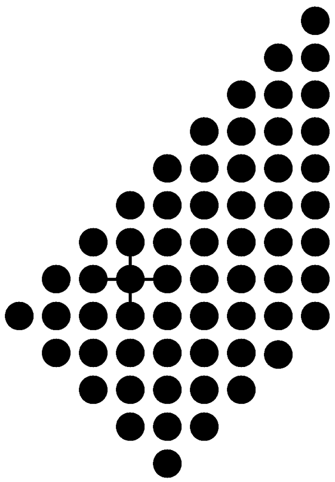
Y_3 = 

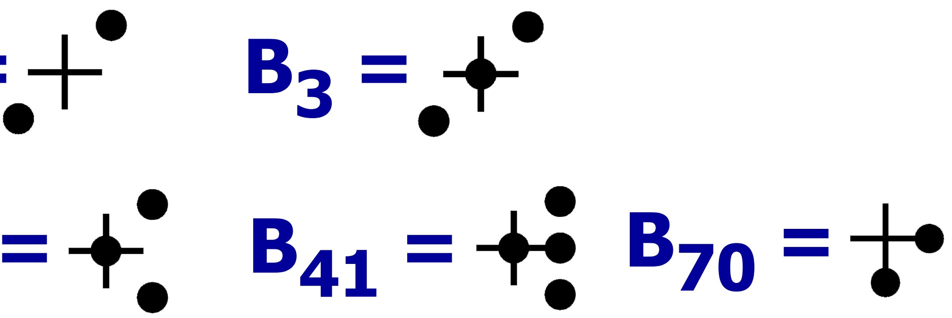
$S_3[A]=[B_2, B_3, B_{40}]$

$\rho(Y_3) = (3,6)$

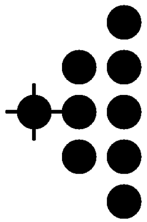
$\rho(Y_3) \neq (0,0)$? (Sim)

B_{40} é invariante de Y_3? (Sim)

$A =$ 



$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

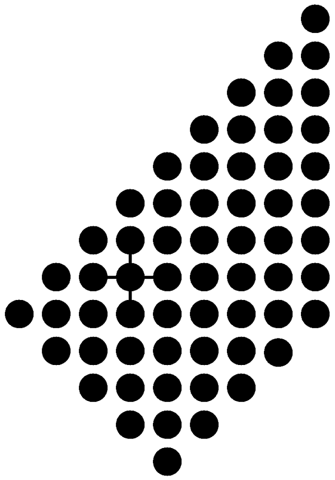
$Y_4 =$ 

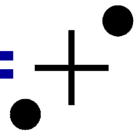
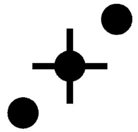
$S_4[A]=[B_2, B_3, B_{40}, B_{40}]$

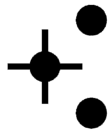
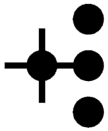
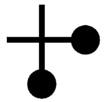
$\rho(Y_4) = (2,4)$

$\rho(Y_4) \neq (0,0)$? **(Sim)**

B_{40} é invariante de Y_4? **(Sim)**

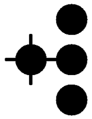
$A =$ 

$B_2 =$  $B_3 =$ 

$B_{40} =$  $B_{41} =$  $B_{70} =$ 

$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

↑

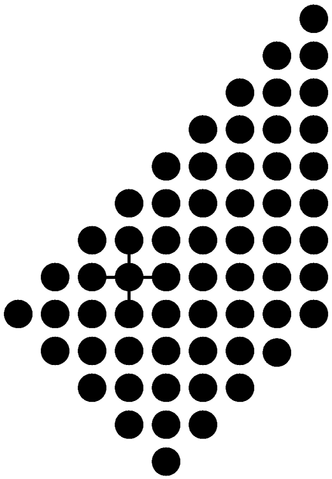
$Y_5 =$ 

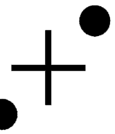
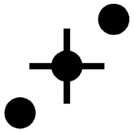
$S_5[A]=[B_2, B_3, B_{40}, B_{40}, B_{40}]$

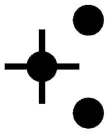
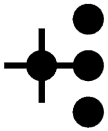
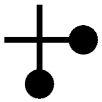
$\rho(Y_5) = (1,2)$

$\rho(Y_5) \neq (0,0)$? **(Sim)**

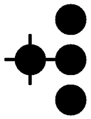
B_{40} é invariante de Y_5? **(Não)**

A = 

B_2 =  B_3 = 

B_{40} =  B_{41} =  B_{70} = 

SeqQ=[B_1, B_2, B_3, - - - , B_{40}, B_{41}, - - - ,B_{70} , - - -]

Y_5 = 

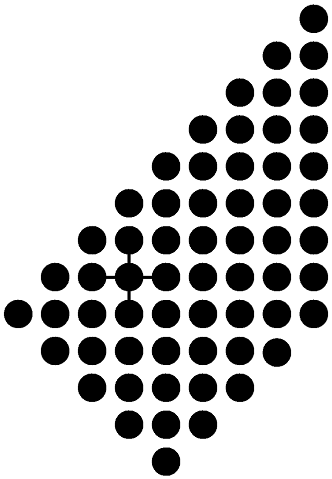
S_5[A]=[B_2, B_3 , B_{40} , B_{40} , B_{40}]

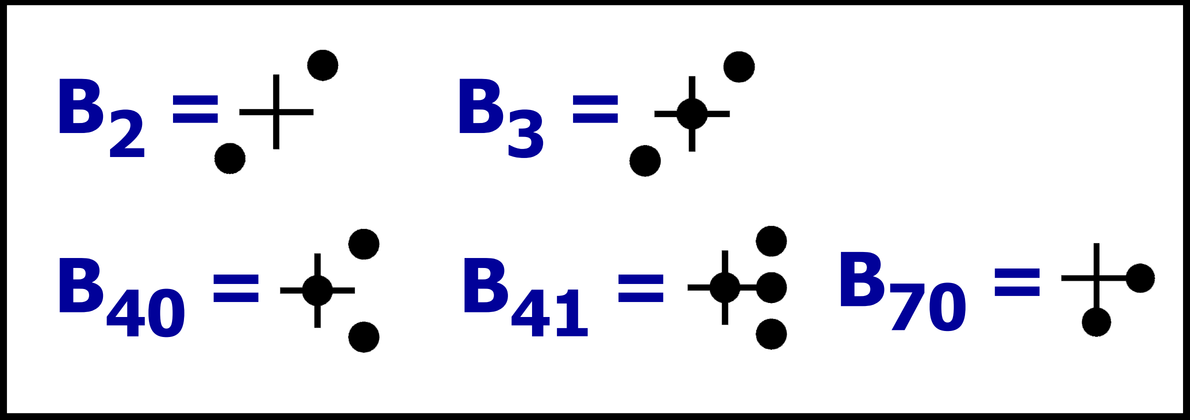
$\rho(Y_5) = (1,2)$

$\rho(Y_5) \neq (0,0)$? **(Sim)**

B_{41} é invariante de Y_5? **(Sim)**

A =





$SeqQ=[B_1, B_2, B_3, \cdots, B_{40}, B_{41}, \cdots, B_{70}, \cdots]$

$Y_6 =$

$S_6[A]=[B_2, B_3, B_{40}, B_{40}, B_{40}, B_{41}]$

$\rho(Y_6) = (0,0)$

$\rho(Y_6) \neq (0,0)$? (Não)

Decomposição de EE´s

EE´s Simplesmente Conexos

EE´s Simplesmente Conexos

H. Park & R. T. Chin

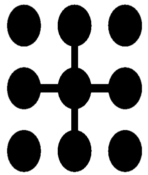
Decomposition of Arbitrarily Shaped Morphological Structuring Elements

1995

EE´s Simplesmente Conexos

Dado um EE A Simplesmente Conexo

Encontrar uma seqüência $[B_1, B_2, \cdots, B_n]$

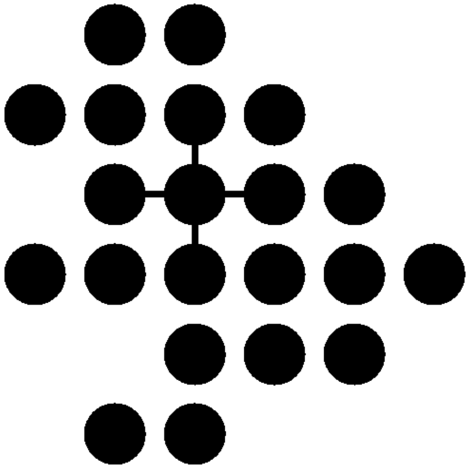
$B_i \subseteq$ 

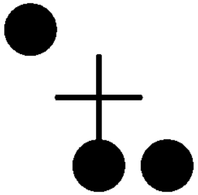
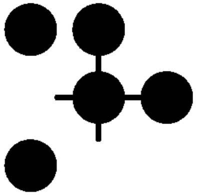
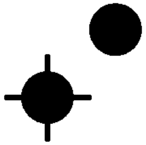
B_i simplesmente conexo

$A = B_1 \oplus B_2 \oplus \cdots \oplus B_n$

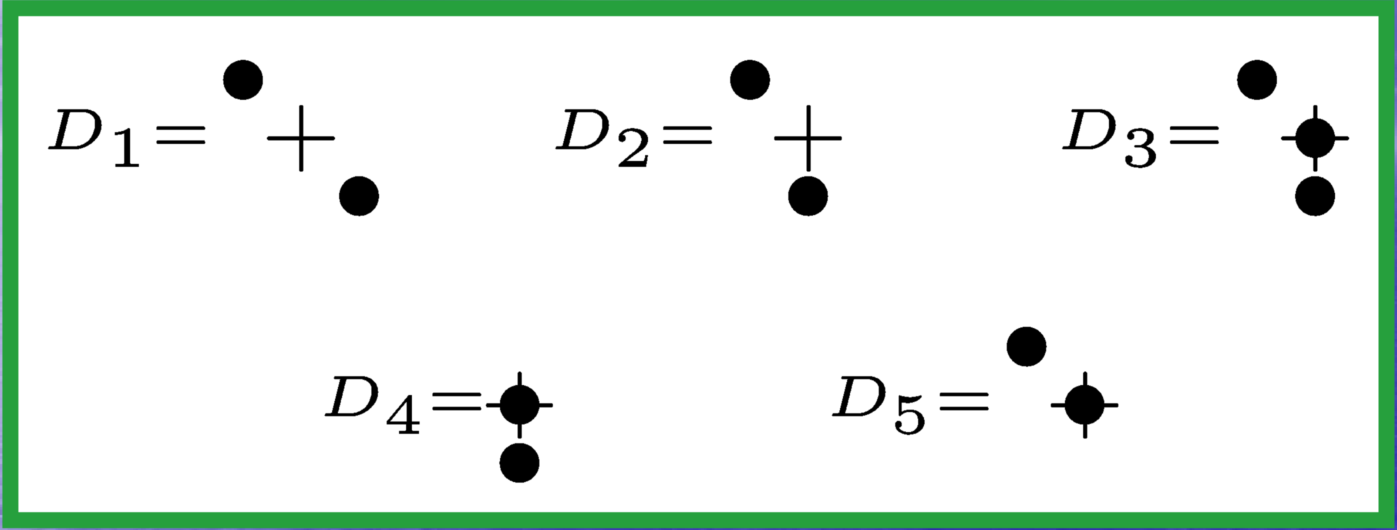
n mínimo

EE´s Simplesmente Conexos

A = 

$C_1 =$  $C_2 =$  $C_3 =$ 

EE´s Simplesmente Conexos

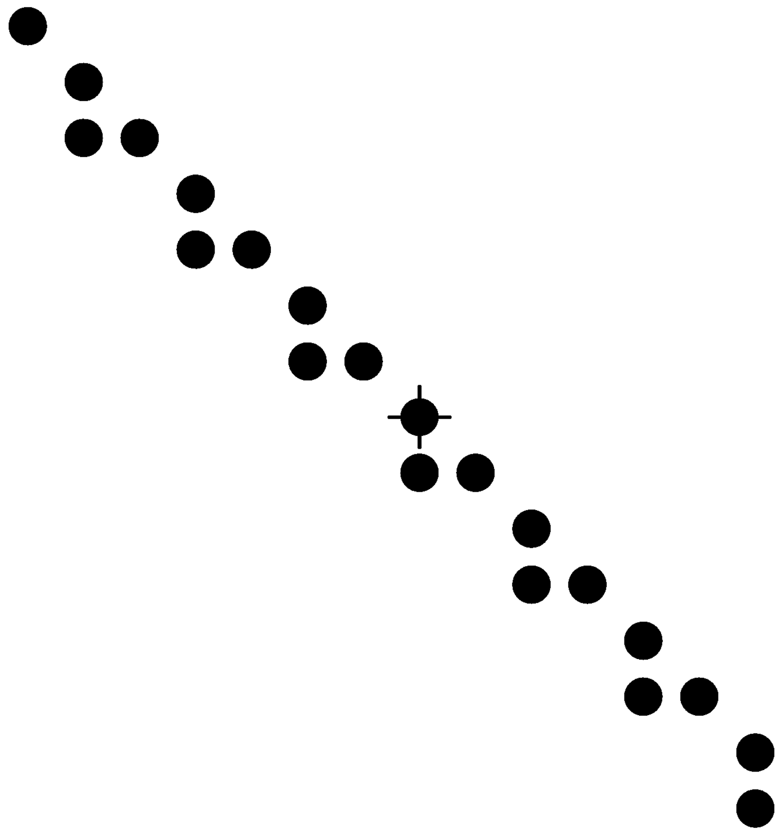


$$X_i = iD_1 \oplus D_3$$
$$Y_i = iD_2 \oplus D_4$$
$$Z_i = iD_2 \oplus D_5$$

$$\mathcal{X} = \{X_i : i > 0\}$$
$$\mathcal{Y} = \{Y_i : i > 0\}$$
$$\mathcal{Z} = \{Z_i : i > 0\}$$

EE´s Simplesmente Conexos

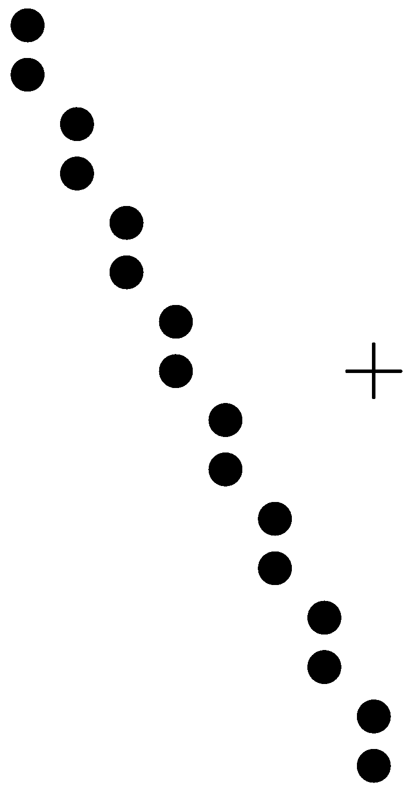
$X_6 =$



$[D_1, D_1, D_1, D_1, D_1, D_1, D_3]$

EE´s Simplesmente Conexos

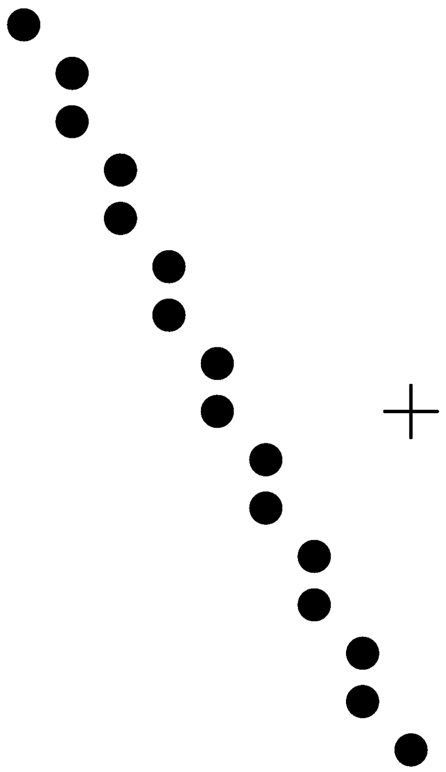
$Y_7 =$



$[D_2, D_2, D_2, D_2, D_2, D_2, D_2, D_4]$

EE´s Simplesmente Conexos

$Z_7 =$



$[D_2, D_2, D_2, D_2, D_2, D_2, D_2, D_5]$

Decomposição de EE´s

EE´s Arbitrários

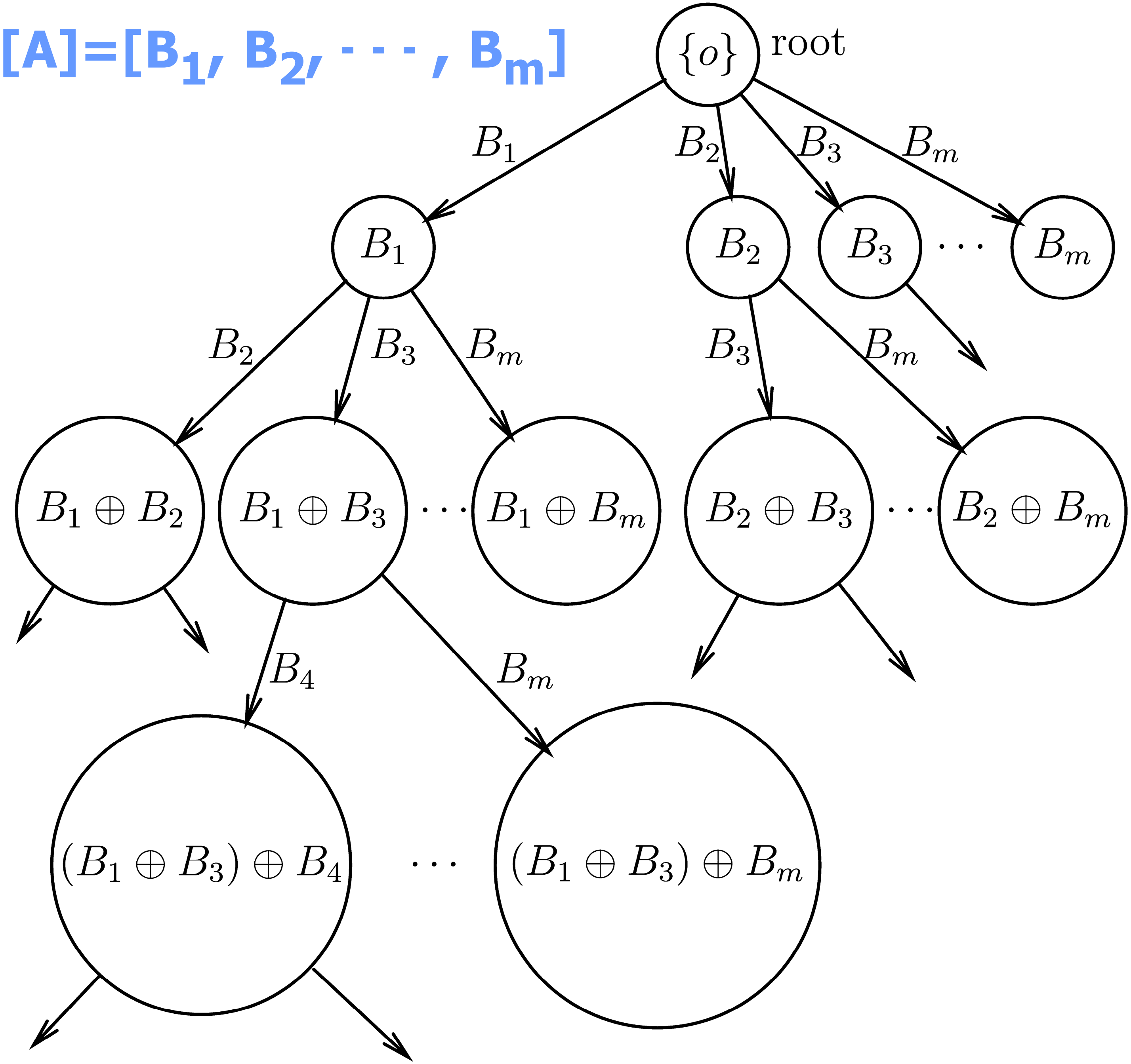
EE´s Arbitrários

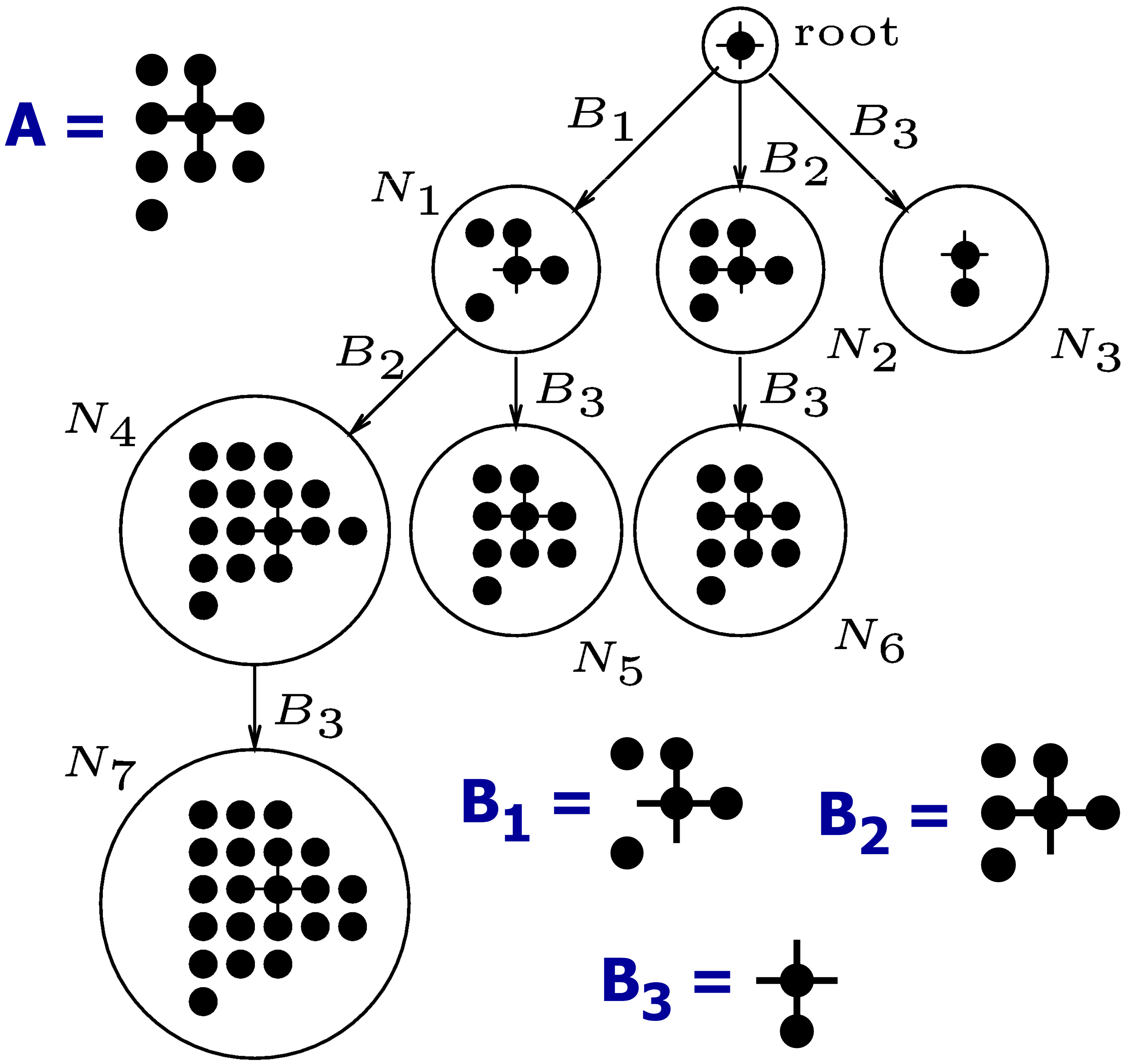
Seqüência de invariantes

SeqInv [A] = subconjuntos do quadrado elementar que são invariantes de A

A tem uma decomposição ⇔ Existe uma subseqüência de SeqInv[A] que é uma decomposição de A

SeqInv[A]=[B_1, B_2, ···, B_m]




A =
root
B1
B2
B3
N1
N2
N3
B2
B3
B3
N4
N5
N6
B3
N7
B1 =
B2 =
B3 =

EE´s Arbitrários

não é possível enumerar em
um tempo razoável
todas subseqüências de SeqInv[A]

Problema de Otimização Combinatória

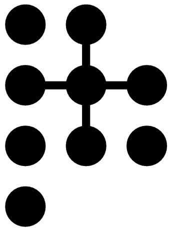
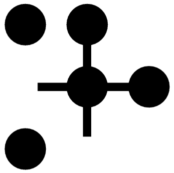
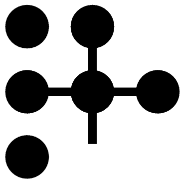
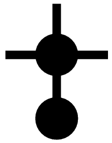
Método de “Branch and Bound”

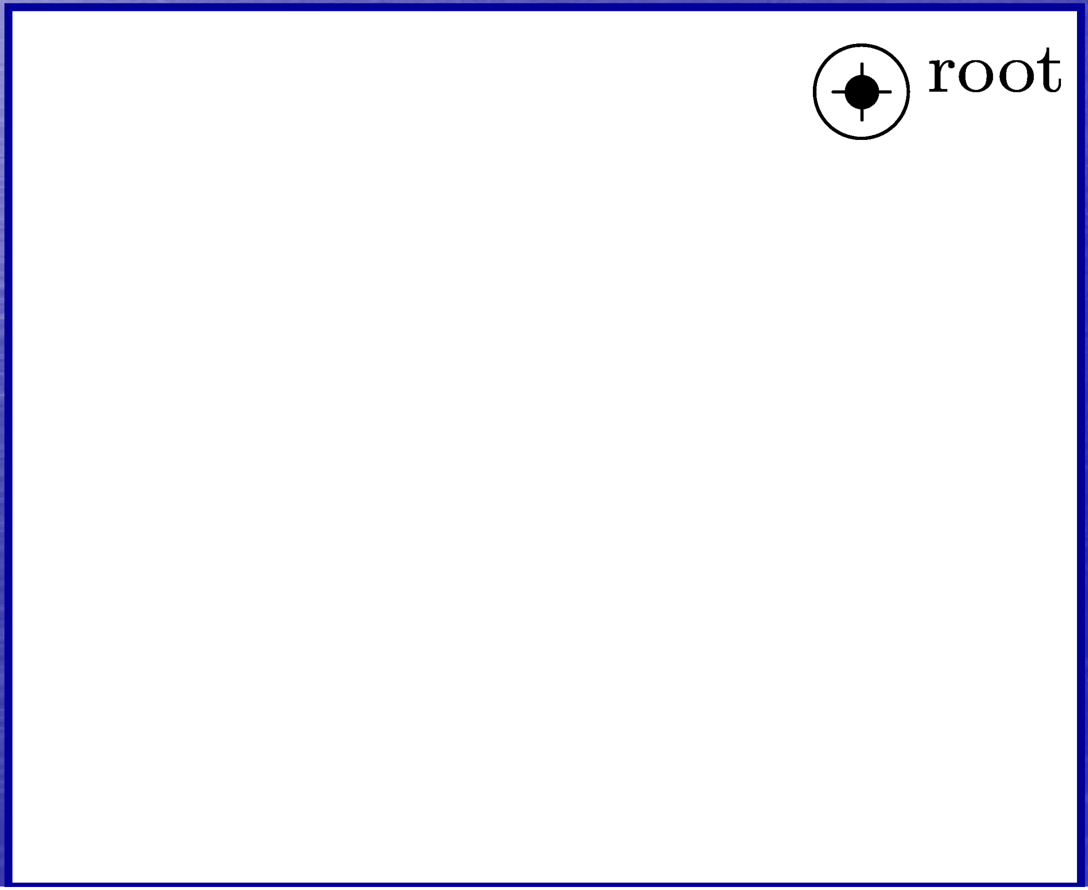
A arte desta estratégia é encontrar boas
podas para evitar de examinar
todas as possibilidades

EE´s Arbitrários

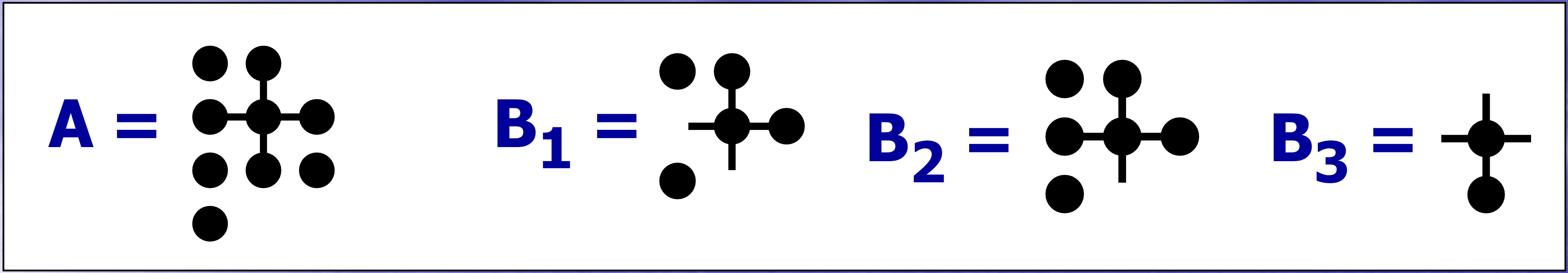
- Poda por Invariança
- Poda por Vetor de Projeção
- Poda por Seqüência Possível

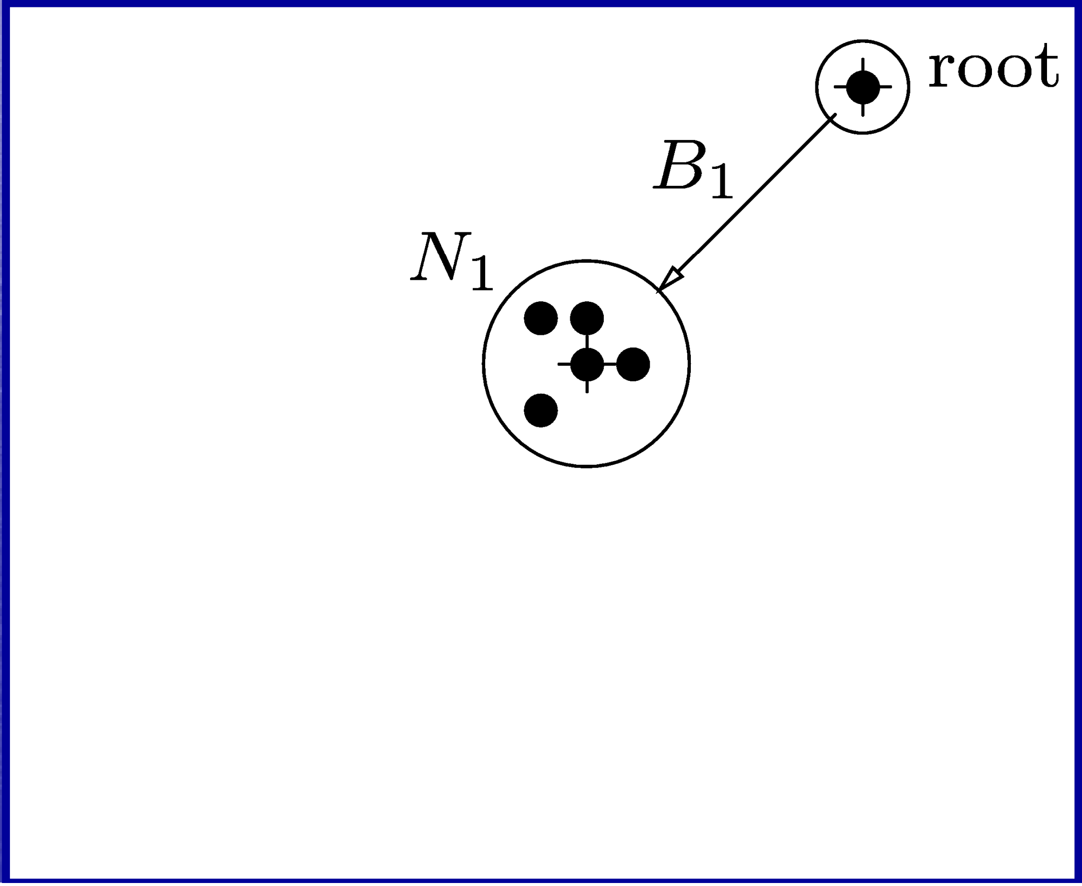
EE´s Arbitrários

$A =$  $B_1 =$  $B_2 =$  $B_3 =$ 

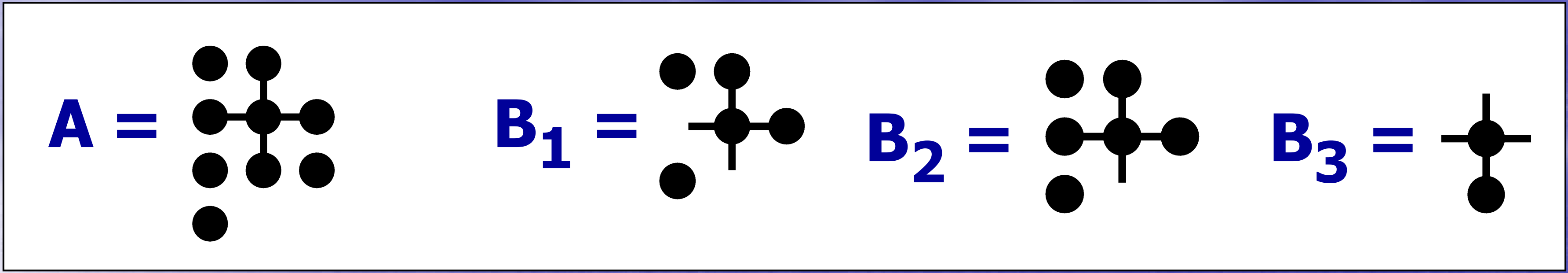


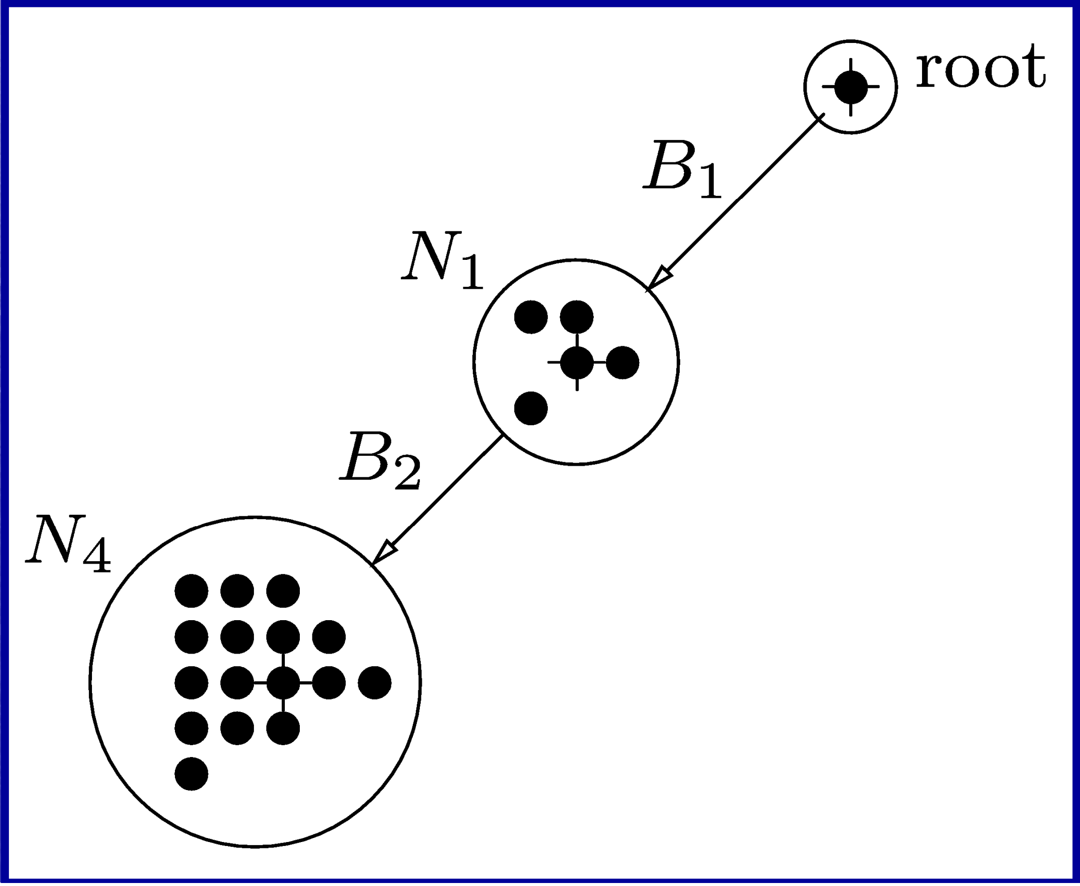
EE´s Arbitrários



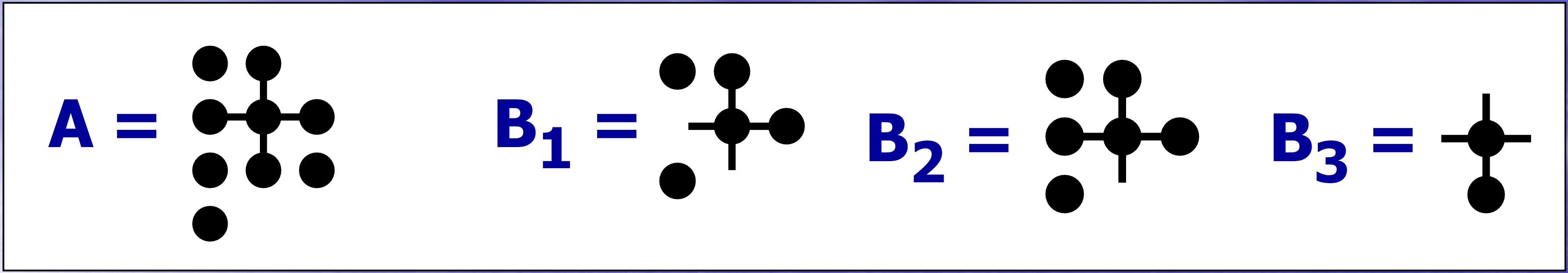


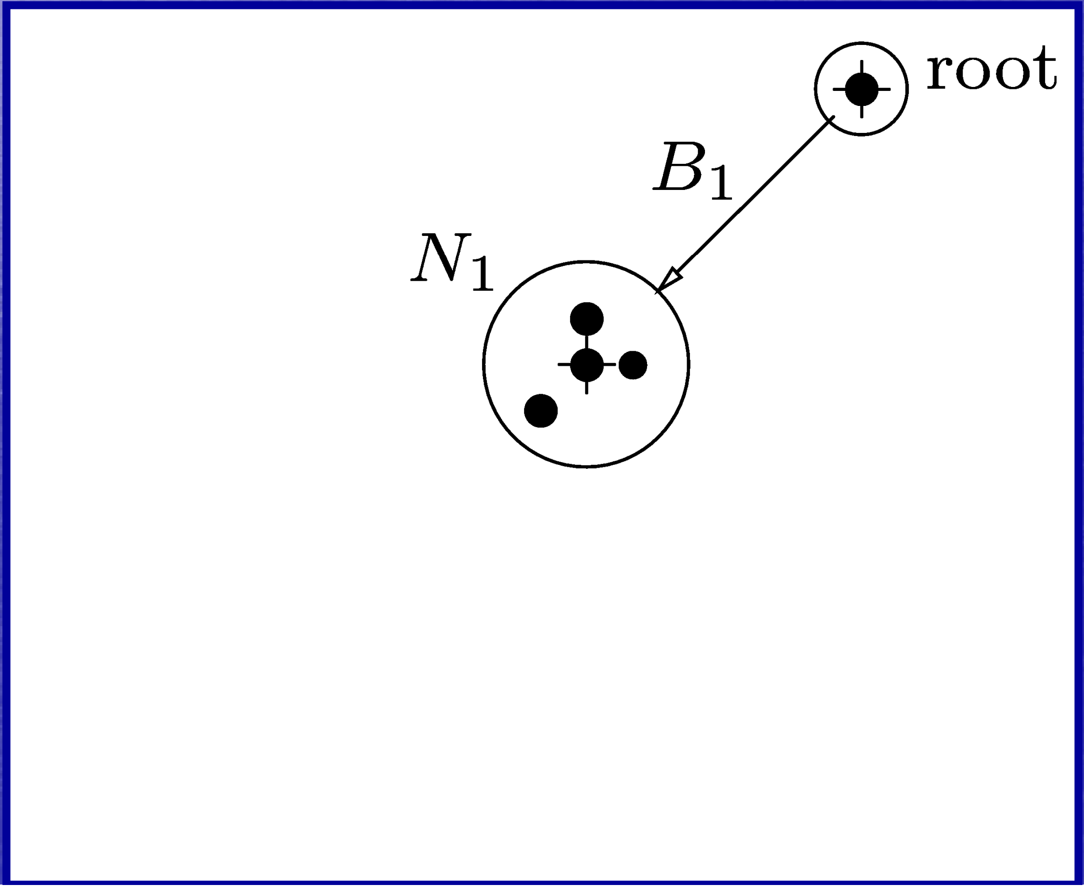
EE´s Arbitrários



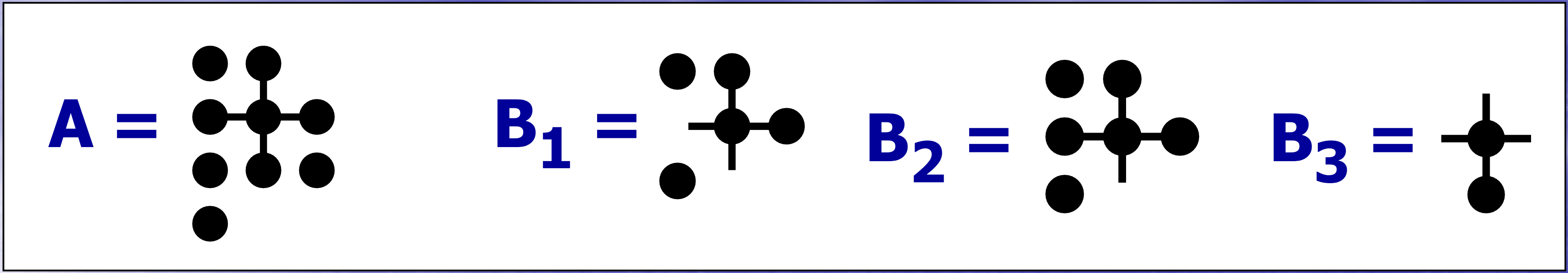


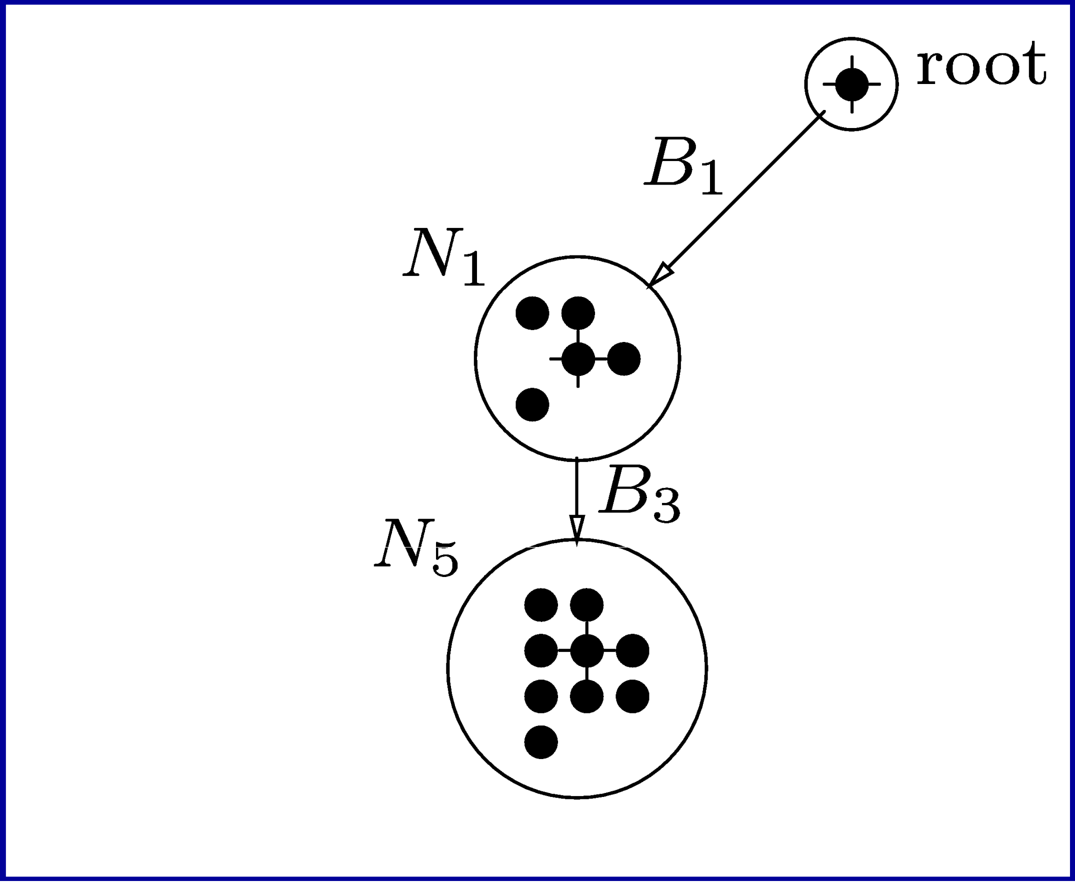
EE´s Arbitrários





EE´s Arbitrários





Mudança de Estrutura

Representação Compacta

Representação Compacta

Se $\psi \in \Psi_W$, então

$$\psi = \bigwedge\{\lambda_{A,B} : [A,B] \in \mathbf{B}(\psi)\}$$

$$\mathbf{E}(\psi) \subseteq \mathbf{B}(\psi)$$

$$C_{[A,B]} \subseteq \mathbf{W}$$

$$\psi = \bigvee\{\delta_{C_{[A,B]}} \lambda_{A,B} : [A,B] \in \mathbf{E}(\psi)\}$$

Representação Compacta

$\psi \in \Psi_W$ **é *autolimitado***

$\forall [A,B] \in \mathrm{B}(\psi)$ $\forall X \in [A,B]$

$\psi(X) \cap W \in [A,B]$

Representação Compacta

$$\psi \in \Psi_W$$

- **Anti-extensivo**
- **Autolimitado**

$$\psi = \bigvee \{\delta_A \lambda_{A,B} : [A, B] \in \mathbf{E}(\psi)\}$$

Representação Compacta

$\psi \in \Psi_W$

Abertura

- **Anti-extensivo**
- **Crescente**
- **Idempotente**

$$\psi = \bigvee\{\delta_A \varepsilon_A : [A, W] \in \mathbf{E}(\psi)\}$$

Representação Compacta

$\psi \in \Psi_W$

Abertura

$$\psi = \bigvee \{\delta_A \varepsilon_A : A \in Inv(\psi)\}$$

A,B $\in Inv(\psi)$

A é invariante de B

$$\psi = \cdots \vee \delta_A \varepsilon_A \vee \cdots \vee \delta_B \varepsilon_B \vee \cdots$$

Representação Compacta

$\psi \in \Psi_W$

Abertura

$$\psi = \bigvee \{\delta_A \varepsilon_A : [A, W] \in \mathbf{E}(\psi)\}$$

$\nexists$**[A,W]** $\in \mathbf{E}(\psi)$

$\nexists$**[B,W]** $\in \mathbf{E}(\psi)$

A é invariante de B

Mudança de Estrutura

Representação Seqüencial

Representação Seqüencial

$$\epsilon_{A_1} \vee \epsilon_{A_2} \vee \cdots \vee \epsilon_{A_n}$$

$$\delta_A \, \epsilon_A \, \epsilon_B \delta_C \, \delta_B \, \epsilon_D$$

Representação Seqüencial

Equação de Fatoração de Minkowski

$$C \in P(E)$$

$$Y_{W'} \in \Pi_{W'}$$

$$W \in P(E)$$

$$X_W \in \Pi_W$$

$$X_W \oplus C^t = Y_{W'}$$

Representação Seqüencial

$$X_W \oplus C^t = Y_{W'}$$

$$W = W' \ominus C^t$$

$$L_W \leq X_W \leq U_W$$

Lower bound

Upper bound

Representação Seqüencial

$C \in P(E)$ é viável

$$\Updownarrow$$

$$X_w \oplus C^t = Y_{w'}$$

tem pelo menos uma solução.

Representação Seqüencial

Algoritmo Search _All

Entrada: $Y_{W'} \in \mathfrak{R}_{W'} \subseteq \Pi_{W'}$

Saída: $(C, X_W) \in P(E) \times \mathfrak{R}_W$

$$X_W \oplus C^t = Y_{W'}$$

$$W = W' \ominus C^t$$

Representação Seqüencial

$$\psi = \delta_{C}\,\varphi \Leftrightarrow B(\psi) = B(\varphi) \oplus C^{t}$$

$$\psi = \epsilon_{C^{t}}\,\varphi \Leftrightarrow B(\psi^{*}) = B(\varphi^{*}) \oplus C^{t}$$

Representação Seqüencial

$$\psi \in \Upsilon_W$$

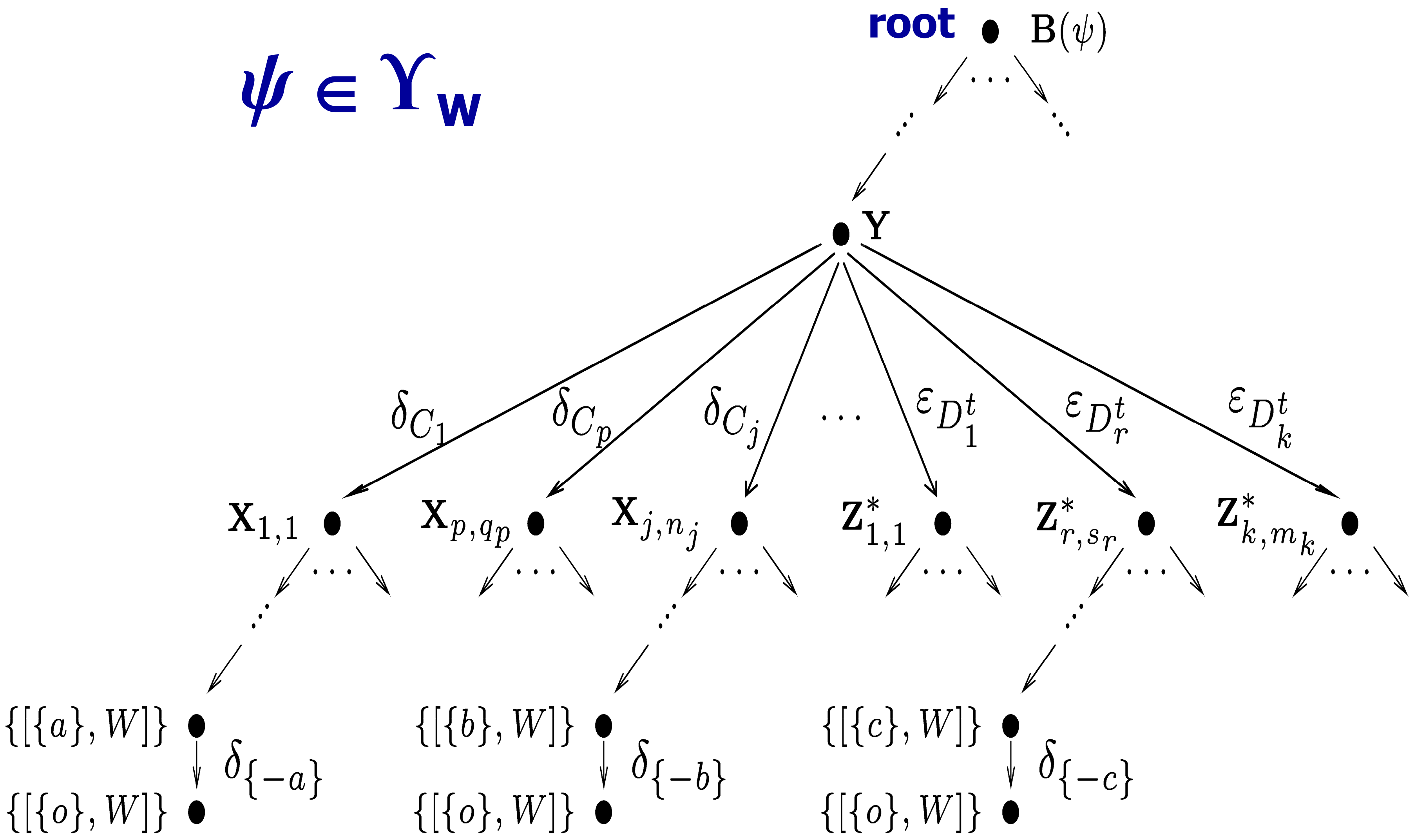
$$\Updownarrow$$

ψ pode ser construído pela composição de erosões e dilatações

$$\psi = \epsilon_A \delta_B \epsilon_B \cdots \epsilon_A \delta_B \delta_A \epsilon_C \delta_C$$

Representação Seqüencial

$\psi \in \Upsilon_{\mathbf{W}}$



Representação Seqüencial

$$\psi \in \Upsilon_W$$

$$Y = B(\psi)$$

Representation Tree

$\bullet\ Y_1 = Y$

$$W = W_1 = 11\underline{1}11$$

$$\mathbf{Y}_1 = \mathbf{Y} = \{[00\underline{1}01], [10\underline{1}00], [01\underline{0}00], [00\underline{0}10]\}$$

$$\psi = \epsilon_P \vee \epsilon_Q \vee \epsilon_R \vee \epsilon_S$$

(a)

Representação Seqüencial

Representation Tree

$$\delta_{C_1}: \bullet\ Y_1 = Y \ \downarrow\ \bullet\ Y_2 = X_1$$

$W = W_1 = 11\underline{1}11$

$\mathbf{Y}_1 = \mathbf{Y} = \{[00\underline{1}01], [10\underline{1}00], [01\underline{0}00], [00\underline{0}10]\}$

$\mathbf{Y}_1^* = \mathbf{Y}^* = \{[11\underline{0}11], [01\underline{1}10]\}$

Output of SEARCH_ALL $(\mathbf{Y}_1)$:

$C_1 = 10\underline{1}$

$\mathbf{Y}_2 = \mathbf{X}_1 = \{[10\underline{1}], [01\underline{0}]\}$

$W_2 = W \ominus C_1^t = 11\underline{1}$

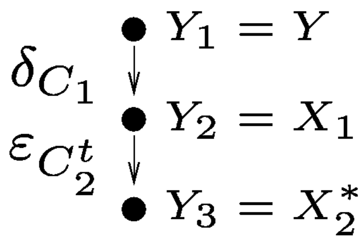
Output of SEARCH_ALL $(\mathbf{Y}_1^*)$:

$\emptyset$.

(b)

Representação Seqüencial

Representation Tree



$W_2 = 11\underline{1}$

$\mathbf{Y}_2 = \{[10\underline{1}], [01\underline{0}]\}$

$\mathbf{Y}_2^* = \{[11\underline{0}], [01\underline{1}]\}$

Output of SEARCH_ALL $(\mathbf{Y}_2)$:

$\emptyset.$

Output of SEARCH_ALL $(\mathbf{Y}_2^*)$:

$C_2 = 1\underline{1}$

$\mathbf{Y}_3^* = \mathbf{X}_2 = \{[11\underline{0}]\}$

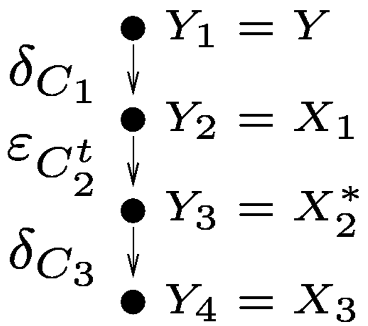
$\mathbf{Y}_3 = \mathbf{X}_2^* = \{[01\underline{0}], [10\underline{0}]\}$

$W_3 = W_2 \ominus C_2^t = 11\underline{0}$

(c)

Representação Seqüencial

Representation Tree



$W_3 = 11\underline{0}$

$\mathbf{Y}_3 = \{[01\underline{0}], [10\underline{0}]\}$

$\mathbf{Y}_3^* = \{[11\underline{0}]\}$

Output of SEARCH_ALL ($\mathbf{Y}_3$):

$C_3 = \underline{1}1$

$\mathbf{Y}_4 = \mathbf{X}_3 = \{[1\underline{0}]\}$

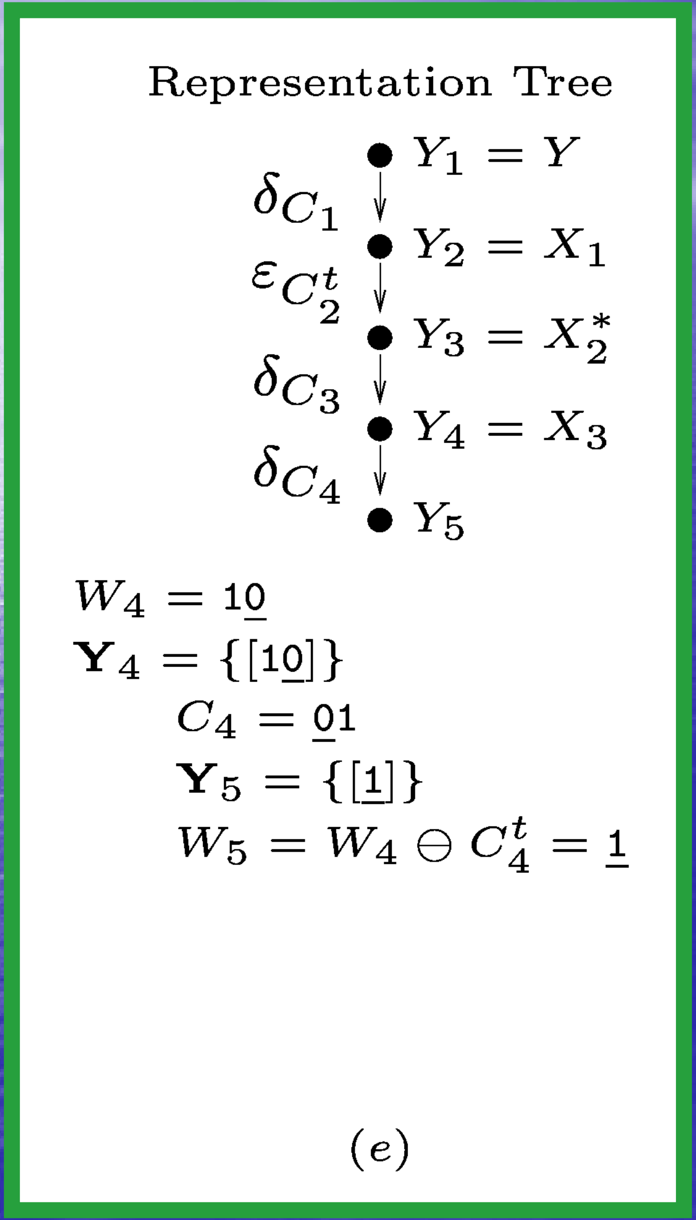
$W_4 = W_3 \ominus C_3^t = 1\underline{0}$

Output of SEARCH_ALL ($\mathbf{Y}_3^*$):

$\emptyset$.

(d)

Representação Seqüencial



$$\psi = \epsilon_P \vee \epsilon_Q \vee \epsilon_R \vee \epsilon_S$$

9 operações

$$\psi = \delta_{C_1} \varepsilon_{C_2^t} \delta_{C_3} \delta_{C_4}$$

7 operações

Representação Seqüencial

Operator	NOSD	NOMS	T
ψ_1	111	30	1.8s
ψ_2	111	16	12.0s
ψ_3	319	17	1.0s
ψ_4	431	20	4.2s
ψ_5	187	15	48m23s
ψ_6	143	16	30m43s

NOSD = Número de operações usando a sup-decomposição.

NOMS = Número de operações usando a melhor solução.

T = Tempo para encontrar a melhor solução.